Cannon-Thurston Maps

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Geometric Structures on Surfaces

1. Differential Geometry: Constant curvature metrics: +1 ($g = 0$), 0 ($g = 1$), -1 ($g \geq 2$).
2. Lie groups: **Discrete faithful** representation $\rho : \pi_1(S) \to PSL(2, \mathbb{R}) = SL(2, \mathbb{R})/\{\pm I\} = Isom^+(\mathbf{H}^2)$.
3. Complex Geometry: Riemann surfaces : transition functions complex analytic.
4. Algebraic Geometry: Solution sets to algebraic equations: (Complex) 1 dimensional smooth varieties in $\mathbb{C}P^n$.

Poincaré-Koebe-Klein uniformization theorem establishes a dictionary between these structures.

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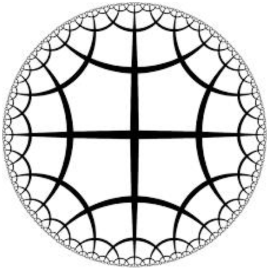
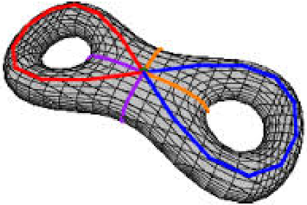
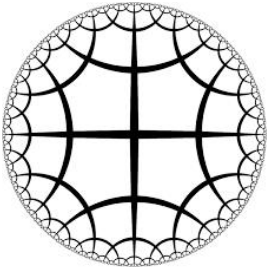


Discrete faithful representation of Coxeter group.



Constant curvature -1 metric on closed surface S of genus at least 2.



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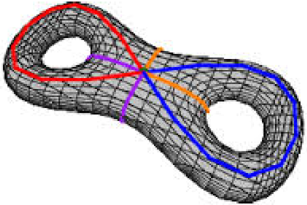

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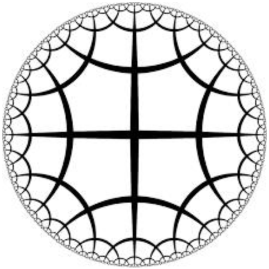

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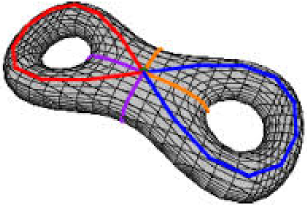

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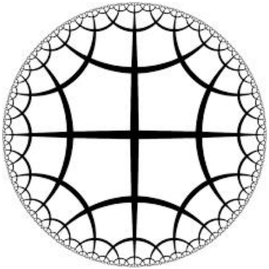

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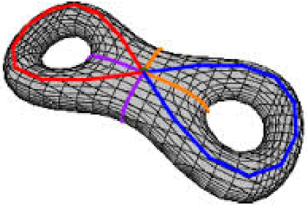

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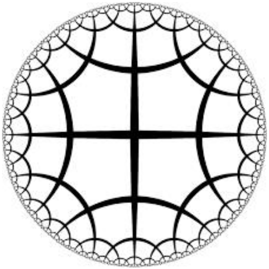

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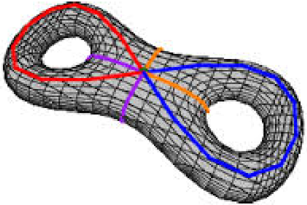

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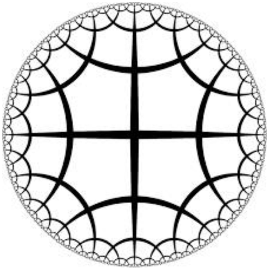

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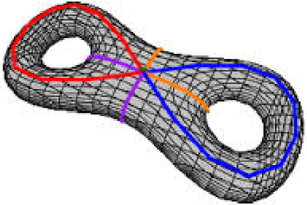

Constant curvature -1 metric on closed surface *S* of genus at least 2.

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Definition (Algebraic topology on space of representations)

A sequence of representations $\rho_n : \pi_1(S) \to PSL_2(\mathbb{C})$ is said to converge **algebraically** to $\rho_\infty : \pi_1(S) \to PSL_2(\mathbb{C})$ if for all $g \in \pi_1(S)$, $\rho_n(g) \to \rho_\infty(g)$ in $PSL_2(\mathbb{C})$.

The collection of conjugacy classes of discrete faithful representations of $\pi_1(S)$ into $PSL_2(\mathbb{C})$ equipped with the algebraic topology is denoted as $AH(S)$

In dimension 2 (for $PSL_2(\mathbb{R})$), this is Teichmüller space.

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Hyperbolic structures on $S \times \mathbb{R}$: Geometry

Theorem (Topological tameness)

(Thurston-Bonahon): *For* $\Gamma = \rho(\pi_1(S))$ *- a Kleinian surface group,* $M = \mathbf{H}^3/\Gamma$ *is homeomorphic to a product* $S \times \mathbb{R}$.

But geometrically, a lot of variety.
So 3-dimensional analog of Teichmüller theory becomes the study of hyperbolic structures on $M = S \times \mathbb{R}$ up to isometry.

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Setup for studying extrinsic geometry

Let $i : S \to M$ be a homotopy equivalence (embedding),
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Dichotomy of representations

Broadly, 3d hyperbolic geometric structures on $S \times \mathbb{R}$ are of two kinds.

1. Quasi-Fuchsian/ Convex cocompact/ undistorted/ quasi-isometrically (qi) embedded:
   Distances in $\mathbf{H}^2$ (denote d_2) and $\mathbf{H}^3$ (denote d_3) are linearly comparable: There exist (k, ϵ) such that
   $$\frac{1}{k} d_2(g.o, h.o) - \epsilon \leq d_3(\rho(g).O, \rho(h).O) \leq k\, d_2(g.o, h.o) + \epsilon.$$
   $QF(S) = Teich(S) \times Teich(S)$ **(Bers' simultaneous uniformization theorem, 1960)** Diagonal=Fuchsian
2. Limits of the above in the algebraic topology. **(Bers' density conjecture proved by Brock-Canary-Minsky, 2012)** These are *not* qi-embedded.

Accordingly, ends of $\mathbf{H}^3/\Gamma = S \times \mathbb{R} = M$ can also be quasi-Fuchsian or degenerate.

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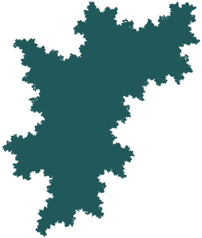

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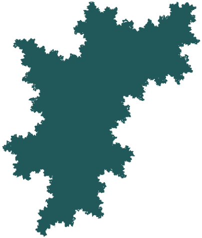

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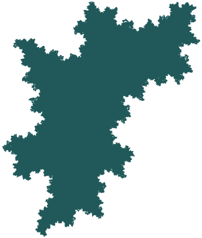

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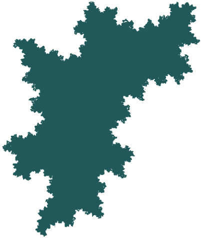

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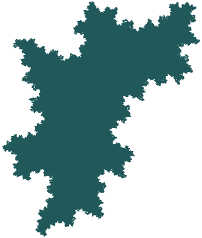

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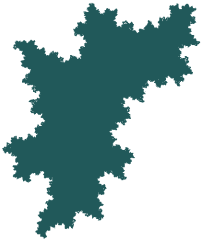

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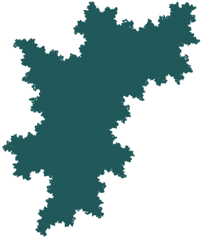

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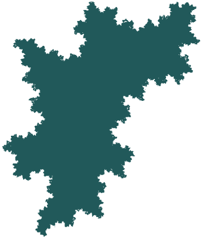

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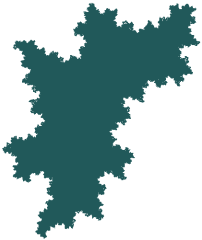

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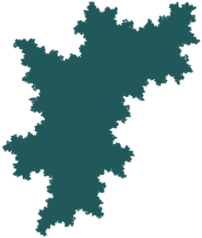

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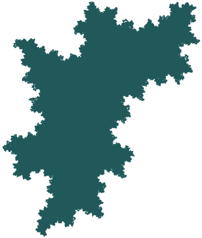

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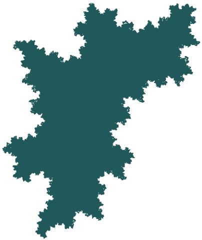

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Jordan curve theorem $\Rightarrow \hat{C} \setminus \Lambda_\Gamma$ = domain of discontinuity = two (topological) disks, quotienting down to Riemann surfaces τ_1, τ_2. (cf. Bers' Simultaneous Uniformization Theorem)

Definition (From complex analytic to geometric perspective:)

The **convex hull** CH_Γ *of* Λ_Γ *is the smallest non-empty closed convex subset of* $\mathbf{H}^3$ *invariant under* Γ.
Let $M = \mathbf{H}^3/\Gamma$. *The quotient of* CH_Γ *by* Γ *is called the convex core* $CC(M)$ *of* M.

For quasi-Fuchsian groups, $CC(M)$ is homeomorphic to $S \times [-1, 1]$.
Limits of quasi-Fuchsian groups:
Thickness of Convex core $CC(M)$ tends to infinity.
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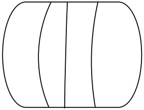
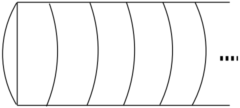
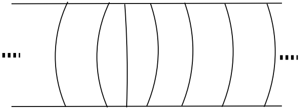


CC(M) compact



CC(M) for simply degenerate



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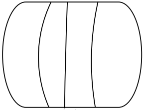
Limits exist as thickness of convex cores tends to infinity.

Simultaneous uniformization perspective for the limits = ??



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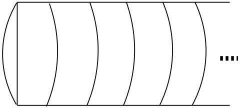

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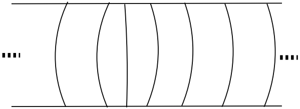

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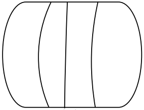

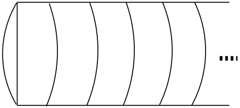

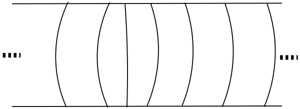

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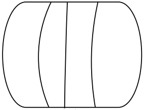

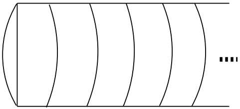

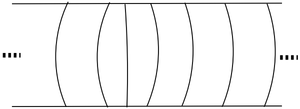

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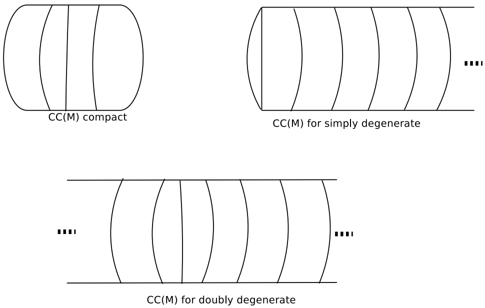

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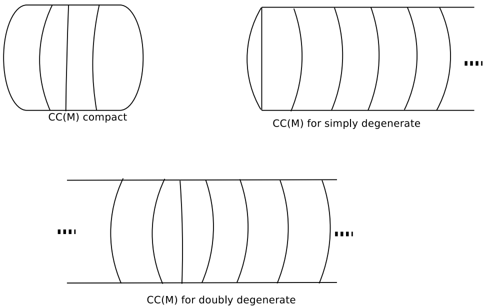

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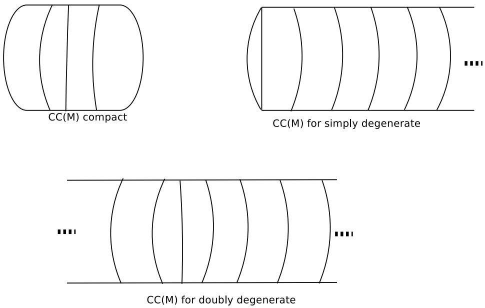

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Cannon-Thurston maps: Asymptotic/dynamic viewpoint for degenerate representations

Theorem (Existence of Cannon-Thurston Maps, 2014, (M–))

For all representations in $AH(S)$, $\tilde{i} : \mathbf{H}^2 \to \mathbf{H}^3$ extends to a **continuous map** $\partial i : S^1 \to S^2$

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For each degenerate end of M, there exists a natural geodesic lamination (a 'laminated object at infinity') called the **ending lamination** $EL(\rho)$ (in place of τ_1, τ_2 for quasi-Fuchsian.)

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Theorem (Local connectivity, 2014, (M–))

Connected limit sets of finitely generated discrete subgroups of $PSL_2(\mathbb{C})$ are locally connected.

Theorem (Arbitrary Kleinian groups, 2017, (M–))

Existence and structure theorem for C-T maps continue to hold for arbitrary finitely generated discrete subgroups of $PSL_2(\mathbb{C})$.

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(IV) Consequences and Generalizations

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Let $H \subset G$ be a hyperbolic subgroup of a hyperbolic group. Does the inclusion $i : \Gamma_H \to \Gamma_G$ of Cayley graphs extend to the boundary, giving $\partial i : \partial H \to \partial G$?

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Geometric group theory

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Higher rank?

Surface group representations in other non-compact semisimple Lie groups G: Anosov representations (Labourie) give $\pi_1(S)-$equivariant embeddings from S^1 to G/P. Analog of $QF(S)$.
Work of Kapovich-Leeb-Porti;
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Question

1. *Analog of elements of limiting representations in $AH(S) \setminus QF(S)$?*
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