On conciseness of words in residually finite groups, Pavel Shumyatsky University of Brasilia pavel2040@gmail.com

A group-word $w$ is called concise if whenever the set of $w$-values in a group $G$ is finite it always follows that the verbal subgroup $w(G)$ is finite. More generally, a word $w$ is said to be concise in a class of groups $X$ if whenever the set of $w$-values is finite for a group $G \in X$, it always follows that $w(G)$ is finite. P. Hall asked whether every word is concise. Due to Ivanov the answer to this problem is known to be negative. On the other hand, for residually finite groups the problem remains wide open. We recently discovered that the Lie-theoretic techniques created by Zelmanov in his solution of the restricted Burnside problem can be used to prove conciseness in residually finite groups of many words. Our talk will be about recent developments with respect to that problem.

Using MIR-max algorithm to monitor performance of donor-funded projects in developing countries based on DHS data, Elias Mwakilama and Martin Paisley Staffordshire University, UK and University of Malawi, Malawi emwakilama@cc.ac.mw

Most developing countries in Africa are faced with problems of ill-health, poverty, human rights violation, and non-access to human growth services such as clean water. As a result, there has been an increase in growth of non-governmental organizations (NGOs) subject to high potential of the donor-funded projects (DFPs) with different commitments such as to reduce poverty and hunger, to remove disparities in gender and economic status, and to improve access to basic services such as education, clean water, justice and health. However, one major concern appearing in the media is lack of progress by such NGOs to tackle the problems either due to mismanagement of donor funds or lack of proper monitoring mechanisms of the project indicators. This paper proposes use of MIR-max algorithm to aid in a process of using such country’s DHS data sets in order to visually monitor performance of donor funded projects. MIR-max is a non-neural Self-organising map (SOM) data exploration method that works well with interval or ordinal data. Exploration of different Malawi DHS data sets is being demonstrated in this paper with MIR-max algorithm in order to expose non-performing
DFPs in Malawi in this era of Big Data.

**Characterization of chaos for piecewise smooth maps**, Alberto A. Pinto
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Let $f$ and $g$ be piecewise smooth interval maps, with critical-singular sets, and $A$ a cycle of intervals for $f$. We prove that $A$ is a topological chaotic attractor if, and only if, $A$ is a metric chaotic attractor. Let $h|A$ be a topological conjugacy between $f$ and $g$. We prove that, if $h$ is differentiable at a single point $p$ of the visiting set $V$, with non zero derivative, then $h$ is smooth in $A$. Furthermore, the visiting set $V$ is a residual set of $A$ and, if the sets $C_f$ and $C_g$ are critical then $V$ has $\mu$ full measure, for every expanding measure $\mu$, with supp $\mu = A$.

**On Switched Dynamical Systems**, Hassane Bouzahir

This work is devoted to stochastic stability analysis of hybrid dynamical systems with Markovian switching, using Lypunov method, $M$-matrix theory and stochastic analysis.

The first part focuses on the stochastic stability of a class of hybrid systems with Markovian switching. We are interested in mean exponential stability of a Markovian jump linear system. We establish the main direct conditions on the subsystem matrices.

The second part is devoted to asymptotic almost sure stability for a hybrid system with a Markovian switched controller. We suppose sufficient conditions on a state space realization of an associated transfer matrix. These conditions guarantee stability through existence of a Lyapunov function.

In the third part, we establish existence and uniqueness of solutions to stochastic functional differential equations with infinite delay in a concrete fading memory phase space. The result therein is used to show existence and uniqueness of solutions and global asymptotic stability of a stochastic neural networks with infinite delay and Markovian switching.

Our work contains examples to demonstrate the applicability and effectiveness of our results.
We deal with the method of multiple shooting (an approximate method) for solving efficiently geometric shortest constrained path problems in 2D and 3D.

The geometric shortest path problem, namely computing the shortest path between two points in a geometric closed region is one of most well-known problems in computational geometry and has many applications. The method of multiple shooting is introduced for the geometric shortest path problem. It consists of three factors:

(f1) Partition of the region into suitable subdomains.

(f2) Collinear/Straightness condition is established at each shooting point.

(f3) The algorithm enforces (f2) at all shooting points, namely all-at-once, when the path is formed by shooting points. Otherwise, an update of shooting points makes the paths better.


In this talk, we address the geometric shortest constrained path problems: computing the shortest descending paths (or shortest gentle paths/shortest descending gentle paths) between two points on a terrain (that were introduced and have been investigated by de Berg and van Kreveld (in 1997), Ahmed and Lubiw (in 2009), Liu and Wong (in 2011), Nöllenburg and Sautter (in 2014)), etc. To date, no exact algorithms are found.

For each case, corresponding factors (f1)-(f3) are constructed and then new algorithm is presented. The method of multiple shooting relies on determining the shortest constrained paths along a triangulated sequence on the terrain. Some advantages of the method are: The method does not rely on Steiner point technique and we do not need graph tools on entire terrains. Our new algorithms are implemented in C++/CGAL and/or visualized by CGAL/JavaView.
Plausible reasoning and the analogy in the creation and solution of a problem for a mathematics competition, Mauro Misael Garcia Pupo University of Antonio Narino mauro@uan.edu.co

When discussing how is the mathematical thinking in the process of construction of a problem, in particular of the magic square generator, it is being investigated in the so-called “Metacognition”. The same problem is applied to a group of 32 students in a training of the Colombian Math Olympics in 2016 as a qualifying test and analyzed all solutions; as well as the associated cognitive processes. Identified that the both processes lead to contrasted sensibility. It is very interesting to show as a plausible reasoning presented, perpetually, in the process of creating the problem and analogy in all the solutions.

Converting Graphic Relationships into Conditional Probabilities in Bayesian Network, Loc Nguyen ng_phloc@yahoo.com

Bayesian network is a powerful mathematical tool for prediction and diagnosis applications. A large Bayesian network can be constituted of many simple networks which in turn are constructed from simple graphs. A simple graph consists of one child node and many parent nodes. The strength of each relationship between a child node and a parent node is quantified by a weight and all relationships share the same semantics such as prerequisite, diagnostic, and aggregation. In the first goal, the research focuses on converting graphic relationships into conditional probabilities in order to construct a simple Bayesian network from a graph. Relationship conversion is adhered to logic gates such as AND, OR, and XOR, which is essential feature of the research. Especially, SIGMA gate is introduced to convert aggregation relationship into conditional probabilities of Bayesian network for applications of diagnosis and assessment. Note that the SIGMA gate inference was discovered by two authors Eva Millán and José Luis Pérez-de-la-Cruz in their research paper “A Bayesian Diagnostic Algorithm for Student Modeling and its Evaluation” in 2002. Because diagnostic relationship is the important subject, the second goal of this research is to propose the diagnostic condition for validating a relationship conversion within context of diagnostic application. The
diagnostic theorem is proposed and proved, which gives facilities for testing such diagnostic condition.

**Zariski cancellation problem for noncommutative rings**, Oswaldo Lezama
*Universidad Nacional de Colombia, Bogotá*

The Zariski cancellation problem arises in commutative algebra and can be formulated in the following way: Let $K$ be a field and $B$ be a commutative $K$-algebra,

$$\text{if } (K[t_1, \ldots, t_n])[t] \cong B[t], \text{ then } K[t_1, \ldots, t_n] \cong B?$$

Recently the problem has been studied for noncommutative algebras that are domains (see for example, Bell, J., and Zhang, J. J., *Zariski cancellation problem for noncommutative algebras*, Selecta Math. (N.S.) 23 (2017), no. 3, 1709–1737). In this short talk we consider the Zariski cancellation problem for noncommutative rings that are not necessarily domains.

**Inverse source non-local problem for mixed type equation with Caputo fractional differential operator**, E. Karimov, N. Al-Salti and S. Kerbal
*Academy of Sciences of the Republic of Uzbekistan and Sultan Qaboos University*

We consider the unique solvability of an inverse-source problem with integral transmitting condition for a time-fractional mixed type equation in rectangular domain where the unknown source term depends only on the space variable. The solution is based on a series expansion using a bi-orthogonal basis in space, corresponding to a non-self-adjoint boundary value problem. Under certain regularity conditions on the given data, we prove the uniqueness and existence of the solution for the given problem. The influence of the transmitting condition on the solvability of the problem is also demonstrated. Two different transmitting conditions are considered — viz. a full integral form and a special case. In order to simplify the bulky expressions appearing in the proof of our main result, we establish a new property of the recently introduced Mittag-Leffler type function in two variables.

**Approximation of closed convex set by semidefinite representable set**, 5
The work deals with the technique of approximating any closed convex set by \textbf{semidefinite representable set}. Semidefinite representable sets are the feasible regions of \textbf{semidefinite programming problems}. Victor Klee, the father of \textit{Convex Analysis} proved in 1959 that any closed convex set can be approximated by \textit{boundedly polyhedral set} as well as by \textit{polyhedral set}. As polyhedral sets are the feasible regions of linear programming problems and semidefinite programming problems generalize linear programming problems, it draws considerable attention to generalize Klee’s approximation technique considering \textbf{semidefinite representable set}. A set is called \textit{boundedly polyhedral set} provided its intersection with each bounded polyhedral set is polyhedral. We consider the general version of boundedly polyhedral set by \textit{compactly semidefinite representable set}. A subset is called \textit{compactly semidefinite representable set} if its intersection with each compact semidefinite representable set is semidefinite representable. So, we extend Klee’s results of the approximation technique of any closed convex set in Euclidean space considering a more general set termed as \textit{compactly semidefinite representable set}. We prove that \textit{there exists a sequence of compately semidefinite representable sets which give more tighter approximation of convex set gradually}. We show that \textit{the sequence strongly converges to the closed convex set}. Further, we develop a technique which gives approximation of closed convex set by \textbf{semidefinite representable set} whereas the approximation is \textit{uniform}. There exists numerous efficient algorithms to solve semidefinite programming problems. Hence, the approximation of closed convex set by semidefinite representable set plays an important role in modern convex optimization.

\textbf{Duadic negacyclic codes over a finite non-chain ring}, Mokshi Goyal and Madhu Raka \textit{Panjab University}

Let \( f(u) \) be a polynomial of degree \( m, m \geq 2 \), which splits into distinct linear factors over a finite field \( \mathbb{F}_q \). Let \( \mathcal{R} = \mathbb{F}_q[u]/\langle f(u) \rangle \) be a finite non-chain ring. In this short communication, we study duadic cyclic codes, duadic negacyclic codes of Type I and Type II over the ring \( \mathcal{R} \) and their extensions. A Gray map from \( \mathcal{R}^n \) to \( (\mathbb{F}_q^m)^n \) is defined which preserves self duality of linear codes. As a consequence,
self-dual, isodual, self-orthogonal and complementary dual (LCD) codes over $\mathbb{F}_q$ are constructed. Some examples are also given to illustrate this.

**Non-existence of wave operators for repulsive Hamiltonians**, Atsuhide Ishida
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We consider the quantum systems described by the Schrödinger equation equipped with so-called repulsive part. In this quantum system, we can see the characteristic property in which the free dynamics of the particle disperse in an exponential order in time. I will report in this talk that we can find a counter example of the slow decaying interaction potential such that the wave operators do not exist and we come to conclusion of the borderline between the short-range and long-range.

**Blocking sets of tangent and external lines to a hyperbolic quadric in PG(3,q)**, Bart De Bruyn, Binod Kumar Sahoo and Bikramaditya Sahu  Bart.DeBruyn@Ugent.be

Let $\mathcal{X}$ be a point-line geometry and $L$ a given nonempty subset of the line set of $\mathcal{X}$. A set $B$ of points of $\mathcal{X}$ is called an $L$-blocking set if each line of $L$ contains at least one point of $B$. Consider $\mathcal{X} = PG(3, q)$, where $q$ is a prime power, and $\mathcal{H}$ a hyperbolic quadric in $PG(3, q)$. Let $\mathcal{E}$ (respectively, $\mathcal{T}$) denote the set of all lines of $PG(3, q)$ which are external (respectively, tangent) to $\mathcal{H}$. We characterize the minimum size ($\mathcal{E} \cup \mathcal{T}$)-blocking sets in $PG(3, q)$ for all $q$.

**On quasi-conformal (in-) compatibility of satellite copies of the Mandelbrot set: I**, Luciana Luna Anna Lomonaco and Carsten L. Petersen lluna@ime.usp.br

The Mandelbrot set is a fractal which classifies the behaviour of the quadratic polynomials $P_c(z) = z^2 + c$. Although it has a remarkably simple definition:

$$\mathcal{M} = \{c \in \mathbb{C} \mid P_c(0) \not\to \infty \text{ as } n \to \infty\}$$
it is a central object in complex dynamics, intriguing and not fully understood. A fascinating fact is the presence of 'baby' Mandelbrot sets in the Mandelbrot set and in many other parameter planes. This peculiarity is explained by the theory of polynomial-like maps developed by Douady and Hubbard. There exist two different types of 'baby' Mandelbrot sets within the Mandelbrot set: the primitive copies, which are visually identical to the Mandelbrot set and have a root cusp, and the satellite ones, without a root cusp. Lyubich proved that for any copy \( M' \), the Douady-Hubbard map \( \chi : M' \to \mathcal{M} \) is a quasiconformal homeomorphism if \( M' \) is primitive, and it is a quasiconformal homeomorphism outside any small neighbourhood of the root if \( M' \) is satellite.

The satellite copies cannot be quasiconformally homeomorphic to the Mandelbrot set (because we cannot straighten a cusp quasiconformally), but they were believed to be mutually quasiconformally homeomorphic (a belief which was enforced by the first author’s proof that the root dynamics of any two satellite copies have quasiconformally conjugate restrictions). Surprisingly, they are not.

Indeed, call \( M_{p/q} \) the satellite copy attached to the main component of the Mandelbrot at the parameter \( c \), where \( P_c \) has a fixed point with multiplier \( \lambda = e^{2\pi i p/q} \) (\( c \) is called the 'root' of the satellite copy), we prove (see On quasi-conformal (in-) compatibility of satellite copies of the Mandelbrot set: I, Invent. Math. 210 (2017), no. 2, 615–644):

**Theorem** For \( p/q \) and \( P/Q \) irreducible rationals with \( q \neq Q \),

\[
\xi := \chi_{P/Q}^{-1} \circ \chi_{p/q} : M_{p/q} \to M_{P/Q}
\]

is not quasi-conformal, i.e. it does not admit a quasi-conformal extension to any neighborhood of the root.

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**Mathematical analysis of a fluid-particle interaction model**, Gabriela Planas and Cristyan Pinheiro *Universidade Estadual de Campinas* gplanas@ime.unicamp.br

We investigate the interaction of a thin spray of particles with a Newtonian, viscous, and incompressible fluid. The fluid is governed by the alpha-Navier-Stokes equations and the particles are described by a density function in the phase space, which solves a Vlasov-like equation. The equations for the fluid and the
spray are coupled through a drag force, which depends on the relative velocity of the fluid and the particles, and on the density function. For the system of alpha-Navier-Stokes-Vlasov we prove existence and regularity of global in-time weak solutions. The convergence of its solutions to that of the Navier-Stokes-Vlasov equations when the parameter alpha tends to zero is also established.

Strategic Placement of Landmarks for Parking and Obstacle Avoidances in a Partially Known Environment, Avinesh Prasad, Bibhya Sharma and Jito Vanualailai University of the South Pacific, Fiji avinesh.prasad@usp.ac.fj

In this paper, we propose a suitable solution for parking and obstacle avoidance of a car-like mobile robot. Our new method is based on strategically fixing landmarks in a bounded workspace that will aid the robot to safely navigate from an initial position to a goal position and finally park correctly inside the designated parking bay. By autonomously controlling the translational velocity and the steering angle, the car-like robot is able to navigate from one landmark to another and finally converge to a target with a pre-defined posture. Computer simulations are used to verify the effectiveness of the proposed method.

Enumeration of self-dual, self-orthogonal and complementary-dual cyclic additive codes, Anuradha Sharma and Taranjot Kaur anuradha@iiitd.ac.in

Let $\mathbb{F}_q$ denote the finite field of order $q$ and characteristic $p$, $n$ be a positive integer coprime to $q$ and $t \geq 2$ be an integer. A cyclic additive code $C$ of length $n$ is defined as an $\mathbb{F}_q$-linear subspace of $\mathbb{F}_{q^t}^n$ satisfying the following property: $(c_0, c_1, c_2, \cdots, c_{n-1}) \in C$ implies that $(c_{n-1}, c_0, c_1, \cdots, c_{n-2}) \in C$. These codes form an important class of error-correcting codes due to their rich algebraic structure and have nice connections with quantum stabilizer codes. Many authors studied their dual codes with respect to the ordinary and Hermitian trace inner products on $\mathbb{F}_{q^t}^n$.

For any integer $t \geq 2$ satisfying $t \not\equiv 1 \pmod{p}$, we place a new trace bilinear form on $\mathbb{F}_{q^t}^n$, which is called the $\ast$ trace bilinear form and is a generalization of the Hermitian trace inner product on $\mathbb{F}_{q^t}^n$ when $q$ is even and $t = 2$. We observe
that it is a non-degenerate, symmetric bilinear form on \( \mathbb{F}_{q^n} \) for any prime power \( q \) and is alternating when \( q \) is even. We study dual codes of cyclic additive codes of length \( n \) with respect to this bilinear form. We further study and enumerate self-dual, self-orthogonal and complementary-dual cyclic additive codes of length \( n \) by placing *, ordinary and Hermitian trace bilinear forms on \( \mathbb{F}_{q^n} \).

**On the structure and distances of repeated-root constacyclic codes of prime power lengths over finite commutative chain rings**, Tania Sidana and Anuradha Sharma taniai@iiitd.ac.in

In this work, we study algebraic structures of repeated-root constacyclic codes of prime power lengths over finite commutative chain rings. Using these algebraic structures, we obtain Hamming distances, symbol-pair distances, Rosenbloom-Tsfasman distances, and Rosenbloom-Tsfasman weight distributions of these codes. As an application of these results, we also list all MDS constacyclic codes of prime power lengths over finite commutative chain rings with respect to the (i) Hamming metric, (ii) symbol-pair metric and (iii) Rosenbloom-Tsfasman metric.

**Systems of Riemann-Liouville fractional equations with multi-point boundary conditions**, Rodica Luca Tudorache rluca@tudorache.com

We study the system of nonlinear ordinary fractional differential equations

\[
(S) \quad \begin{cases} 
D_{0+}^\alpha u(t) + f(t, u(t), v(t)) = 0, & t \in (0, 1), \\
D_{0+}^\beta v(t) + g(t, u(t), v(t)) = 0, & t \in (0, 1), 
\end{cases}
\]

with the coupled multi-point boundary conditions

\[
(BC) \quad \begin{cases} 
D_{0+}^{p_1} u(t)|_{t=1} = \sum_{i=1}^{N} a_i D_{0+}^{q_1} v(t)|_{t=\xi_i}, \\
D_{0+}^{p_2} v(t)|_{t=1} = \sum_{i=1}^{M} b_i D_{0+}^{q_2} u(t)|_{t=\eta_i},
\end{cases}
\]
where \( \alpha \in (n-1,n], \beta \in (m-1,m], n, m \geq 3, p_1, p_2, q_1, q_2 \in \mathbb{R}, p_1 \in [1,n-2], p_2 \in [1,m-2], q_1 \in [0,p_2], q_2 \in [0,p_1], \xi_i, a_i \in \mathbb{R} \) for all \( i = 1, \ldots, N \) \((N \in \mathbb{N}), 0 < \xi_1 < \cdots < \xi_N < 1, \eta_i, b_i \in \mathbb{R} \) for all \( i = 1, \ldots, M \) \((M \in \mathbb{N}), 0 < \eta_1 < \cdots < \eta_M < 1, \) and \( D_{0+}^k \) denotes the Riemann-Liouville derivative of order \( k \) (for \( k = \alpha, \beta, p_1, q_1, p_2, q_2 \)).

Under sufficient conditions on the functions \( f \) and \( g \), which can be nonsingular or singular at the points \( t = 0 \) and/or \( t = 1 \), we prove the existence and multiplicity of positive solutions of problem \((S) - (BC)\). We use some theorems from the fixed point index theory and the Guo-Krasnosel’skii fixed point theorem.

This is a joint research with J. Henderson (Baylor University, Waco, Texas, USA), which was published in the journal *Applied Mathematics and Computation*, vol.309 (2017), pp.303-323.

**On a mixed fractional boundary value problem**, Assia Guezane-Lakoud and Rabah Khaldi a_guezane@yahoo.fr

In this talk, we discuss the existence of solutions for a boundary value problem involving both left Riemann-Liouville and right Caputo types fractional derivatives. For this, we convert the posed problem to a sum of two integral operators then we apply Krasnoselskii’s fixed point theorem to conclude the existence of nontrivial solutions.

**Spectral uniqueness of bi-invariant metrics on compact Lie groups**, Emilio Lauret emiliolauret@gmail.com

Two compact Riemannian manifolds are called isospectral if their associated Laplace-Beltrami operators have the same spectra. There exist in the literature a considerable amount of pairs and families of non-isometric isospectral Riemannian manifolds. However, it is expected that Riemannian manifolds with very nice geometric attributes are spectrally distinguishable, that is, isospectrality implies isometry for them. This talk concerns the case of Riemannian symmetric spaces.

The above problem is very complicated in full generality, so it is usual to re-
strict the space of metrics. We will consider the question of whether a symmetric space given by a semisimple compact Lie group $G$ endowed with a bi-invariant metric is spectrally distinguishable within the space of left-invariant metrics on $G$. A full answer was known only for 3-dimensional compact Lie groups. We will show that the question is affirmative for every symplectic group $Sp(n)$.

**A universal Kolmogorov bound for asymptotic normality of monochromatic edge counts**, Xiao Fang  *xfang@sta.cuhk.edu.hk*

Let $\{G_n : n \geq 1\}$ be a sequence of simple graphs. Suppose $G_n$ has $m_n$ edges and each vertex of $G_n$ is colored independently and uniformly at random with $c_n$ colors. It is known that as $c_n \to \infty$ and $m_n/c_n \to \infty$, the normalized monochromatic edge counts can be approximated in distribution by the standard normal, regardless of the graph structure. An error bound for the Wasserstein distance in the approximation was obtained as $C\left(1/\sqrt{c_n} + \sqrt{c_n/m_n}\right)$ where $C$ is a universal constant. In this short communication, we strengthen the above results by proving the same bound for the Kolmogorov distance in the approximation. Our proof is by a variation of Stein’s method.

**Stability of solitary waves and scattering of solutions for the regularized ZK equation**, Amin Esfahani  *saesfahani@gmail.com*

In this work we study the regularized Zakharov-Kuznetsov equation which was derived to describe unidirectional propagation of ionic-acoustic waves in magnetized plasma. We show the scattering of global small amplitude solutions of this equation. We also establish that if the solution of the Cauchy problem has a compact support for all times, then this solution vanishes identically. Furthermore we investigate the existence, the spectral properties and the orbital stability of solitary wave solutions of this equation.

**Initial boundary value problem for semilinear parabolic equation with non-local Neumann boundary condition**, Alexander Gladkov  *gladkoval@mail.ru*

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Local existence theorem, comparison principle, blow-up problem, the uniqueness and nonuniqueness of a solution for (1)–(3) are considered. We prove some global existence results. Criteria on this problem which determine whether the solutions blow up in finite time for large or for all nontrivial initial data are also given. Our global existence and blow-up results depend on the behavior of $c(x, t)$ and $k(x, y, t)$ as $t \to \infty$. Moreover, we show that under certain conditions blow-up occurs only on the boundary.

A Simple Loop Dwell Time Approach for Stability of Switched Systems, Nikita Agarwal nagarwal@iiserb.ac.in

A continuous-time switched system is a piecewise continuous dynamical system with finitely many subsystems, and a piecewise constant function, known as the switching signal, which determines the switching of the system between subsystems. A signal is represented by the admissible switching from one subsystem to another, and the times at which these switchings take place. Switched systems have applications in electrical and power grid systems, where the underlying graph structure vary with time. In our study, the switching between subsystems is governed by an underlying digraph. There is a bijection between the subsystems and the vertices of the underlying graph. The system can switch from a subsystem to another if there is a directed edge between the corresponding vertices on the underlying graph. Such systems have been studied in in the literature. Even when all the subsystems are stable, the switched system may be unstable for some switching signal. Moreover, one can construct a signal which can stabilize a switched system with all unstable subsystems. Thus, it is evident that the stability of a switched system not only depends on the properties of subsystems, but also on the switching signal.

We will present sufficient conditions on the successive switching times under which the switched system will be stable. For a switched system will all stable subsystems, we will obtain a lower bound on the total time spent (simple loop dwell time) on each simple loop of the underlying graph to guarantee the stability. Further, when the switched system comprises of some unstable subsystems, we give a constructive mechanism of slow-fast switching for stability.
On Spectral Expansions for the Sturm-Liouville Operator with Degenerate Boundary Conditions, Alexander S. Makin alexmakin@yandex.ru

Consider the Sturm-Liouville equation

$$u'' - q(x)u + \lambda u = 0 \quad (1)$$

with degenerate boundary conditions

$$u'(0) + du'(\pi) = 0, \quad u(0) - du(\pi) = 0, \quad (2)$$

where $d \neq 0$ and $q(x)$ is an arbitrary complex-valued function of class $L_1(0, \pi)$.

In this note, we study the basis properties of the root function system of problem (1), (2). Denote $Q(x) = q(x) - q(\pi - x)$.

**Theorem 1.** If for a number $\rho > 0$

$$\lim_{h \to 0} \frac{1}{h^\rho} \int_{\pi-h}^{\pi} Q(x)dx = \nu,$$

and $\nu \neq 0$, then the system of eigenfunctions and associated functions of problem (1), (2) is complete in the space $L_2(0, \pi)$.

**Theorem 2.** The system of eigenfunctions and associated functions of problem (1), (2) never forms a basis in the space $L_2(0, \pi)$.

On classes of structures axiomatizable by universal d-Horn sentences and universal positive disjunctions, Guillermo Badia and Joao Marcos Department of Knowledge-Based Mathematical Systems, Johannes Kepler Universität Linz, Austria and Departament of Informatics and Applied Mathematics, Federal University of Rio Grande do Norte, Brazil guillebadia89@gmail.com

We provide universal algebraic characterizations (in the sense of not involving any “logical notion”) of some elementary classes of structures whose definitions involve universal d-Horn sentences and universally closed disjunctions of atomic formulas. These include, in particular, the classes of fields, of non-trivial rings, and of connected oriented graphs. The classical example of this kind of characterization result is the HSP theorem, but there are myriad other examples (e.g.,
the characterization of elementary classes using isomorphic images, ultraproducts and ultrapowers due to Keisler and Shelah).

**Simple components of rational group algebras**, Gurmeet K. Bakshi and Gurleen Kaur  
gkbakshi@pu.ac.in and gurleenkaur992gk@gmail.com

Let \( \mathbb{Q}G \) be the rational group algebra of a finite group \( G \). A fundamental problem is to determine the structure of the simple components and the explicit expressions of primitive central idempotents of \( \mathbb{Q}G \). The problem is directly related to the character theory of \( G \) over \( \mathbb{Q} \) which is hard to understand. A new approach to this problem was initiated by Olivieri, del Rio and Simon [Comm. Alg. 32 (2004) 1531-1550] where they introduced the notion of Shoda pairs of \( G \). Since then, the theory of Shoda pairs has been developed and used in the several works. The aim of this talk is to describe a method to construct Shoda pairs of \( G \) using Isaacs’s theory of character triples. This will be done for a large class of monomial groups including abelian-by-supersolvable and subnormally monomial groups. The computation of primitive central idempotents and the structure of simple components of \( \mathbb{Q}G \) for groups in this class will be discussed. Finally, we’ll explain the theory with examples.

**Free algebras and new discrete mathematical functions**, Yuri Movsisyan  
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A super-Boolean algebra is an algebra with hyperidentities of the variety of Boolean algebras. A super-De Morgan algebra is an algebra satisfying hyperidentities of the variety of De Morgan algebras. In this talk we give:

1) a characterization of super-Boolean algebras with two binary and one unary operations;

2) a characterization of super-De Morgan algebras with two binary and one unary operations;

3) a characterization of subdirectly irreducible super-Boolean algebras;
4) a characterization of subdirectly irreducible super-De Morgan algebras;

5) a characterization of free \( n \)-generated super-Boolean algebras with two binary, one unary and two nullary operations;

6) a characterization of free \( n \)-generated super-De Morgan algebras with two binary and one unary operations.

A number of open problems are formulated.

Exponentially Harmonic Maps between Finsler Manifolds, Yuan-Jen Chiang

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Exponentially harmonic maps were first introduced by James Eells and Luc Lemaire in 1990. A book about Finsler geometry by David Bao, Shing-Shen Chern and Zongmin Shen was published by Springer in 2000. In this paper, we study exponentially harmonic maps between Finsler manifolds. We derive the first and second variations of the exponential energy of a smooth map between Finsler manifolds. We show that a non-constant exponentially harmonic map \( f \) from a unit \( m \)-sphere \( S^m (m \geq 3) \) into a Finsler manifold is stable in case \( |df|^2 \geq m - 2 \), and is unstable in case \( |df|^2 < m - 2 \). This paper will appear in *Manuscripta Mathematica*.

Controllability of a Space-Time Fractional Order Parabolic Equation, Ramdas B. Sonawane sonawaneramdas@gmail.com

In this paper, we study the null controllability of a space-time fractional order parabolic equation involving fractional diffusion and Caputo fractional time derivative of orders \( s \in (0, 1) \) and \( \gamma \in (0, 1] \). We prove that the system under consideration is not null controllable. Moreover, we generalize some existing results about the null controllability of fractional order parabolic systems. We introduce the adjoint system of space-time fractional order parabolic equation which is used to study the duality relationship between observability and controllability in Hilbert spaces.
Some algebraic structures of soft multisets, Ahmed I. Isah and Yohanna Tella
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Multiset which is an unordered collection of objects where multiples of objects are admitted is of two approaches: multisets of functional approach and multisets of equivalence relation approach. In this short communication, we developed the theory of Soft multisets using multisets of functional approach, introduced the idea of Soft multisets using multisets of equivalence relation approach and identified their distinctive features. It is shown that some properties holding in the latter does not hold in the former. While a Boolean algebra is established in the latter, only a bounded distributive lattice is obtainable in the former. It is further shown that, as the axioms of contradiction and exclusion fails in the former, it holds in the latter. Some related results are also established in both the approaches to Soft multisets.

Semi-Lagrangian Exponential Integration with application to the rotating shallow water equations, P. S. Peixoto and M. Schreiber

In this paper we propose a novel way to integrate time-evolving partial differential equations that contain nonlinear advection and stiff linear operators, combining exponential integration techniques and semi-Lagrangian methods. The general method formulation is built from the solution of an integration factor problem with respect to the problem written with a material derivative, so that the exponential integration scheme naturally incorporates the nonlinear advection. Semi-Lagrangian techniques are used to treat the dependence of the exponential integrator on the flow trajectories. The formulation is general, as many exponential integration techniques could be combined with different semi-Lagrangian methods. This formulation allows an accurate solution of the linear stiff operator, property inherited by the exponential integration technique, as well as providing accurate representation of the nonlinear advection even with large timestep sizes, property inherited by the use of a semi-Lagrangian method. Aiming for application in weather and climate modelling, we discuss possible combinations of well established exponential integration techniques and state-of-the-art semi-Lagrangian methods used operationally in the application. We show experiments
for the rotating shallow water equations, for which results indicate, from one side, that traditional exponential integration techniques could benefit from this formulation to ensure stabler integration with larger timestep sizes and, and from the application perspective that already use semi-Lagrangian methods, that the exponential treatment could improve the solution of wave-dispersion.

**Solvability of a nonlinear fractional Euler-Lagrange type equation**, Rabah Khaldi and Assia Guezane-Lakoud  

In this talk we study a nonlinear Euler-Lagrange type equation involving both left and right fractional derivatives, with a nonlocal condition by means of lower and upper solutions method. For this purpose, we begin by solving an auxiliary problem by using Laplace transform, then we convert the posed problem to an equivalent right-Caputo fractional differential equation with a vanishing terminal boundary condition. After constructing the lower and upper solutions, we define a sequence of modified problems that we solve by Schauder fixed point theorem. Finally, a numerical example is given to illustrate the obtained results.

**Boundary Value Problems with Oscillating Coefficients for Analytic Functions**, Vakhtang Kokilashvili  

The goal of our talk is to present our recent results dealing with the following topics:

1) Solution of the Riemann-Hilbert problem for analytic (generalized analytic) functions with oscillating coefficients in the class of Cauchy type integrals with densities from grand variable exponent function spaces. We consider the case when the coefficients of the boundary conditions belong to the class of oscillating functions, finely suited to the variable analysis. The latter one is an extension of the well-known Simonenko class introduced in the case of constant exponents.

2) On the base of aforementioned results to determine the wide class of weights ensuring the boundedness of singular integral operators in grand variable exponent Lebesgue spaces. We claim that these weights play more constructive role in this problem than so called Muckenhoupt weights.
On the Rellich Inequality in Variable Exponent Lebesgue Spaces, Alexander Meskhi meskhi@rmi.ge

A Rellich inequality in grand variable exponent Lebesgue spaces with exponents that satisfy a condition of log-Hölder type is proved. The research is carried out jointly with D. E. Edmunds.

Dynamic Performance Evaluation of Finite Capacity Multi-Server Queueing System with Vacations, Ram P. Ghimire rpghimire@ku.edu.np

This investigation deals with the derivations of time–dependent performance indices of multi-server queueing system with the provision of working vacation and breakdown vacation of servers. Units arrive in the system in Poisson fashion and are served exponentially in first-come-first-served (FCFS) service discipline. Main objective of this paper is to find the explicit formulae for number of units in the system as well as in queue in time $t$, time spent in waiting in queue and sojourn time of the system. To show the applicability of the model under study the numerical computations of all the measures of performance have been illustrated. Sensitivity analysis of the model has been made so as to identify the most sensitized parameter.

Graphs encoding the generating properties of a finite group, Cristina Acciarri and Andrea Lucchini c.acciarri@mat.unb.br

The generating graph $\Gamma(G)$ of a finite group $G$ is the graph defined on the non-identity elements of $G$ in such a way that two distinct vertices are connected by an edge if and only if they generate $G$. The graph $\Gamma(G)$ gives interesting information only if $G$ is 2-generated.

We introduce an alternative definition that works in a more general setting and encodes the generating properties of a $d$-generated finite group, for any positive integer $d$. In the talk we will present some results and questions related to the study of these graphs. This is a joint work with Andrea Lucchini.
Algebraic Geometry for schemes over symmetric monoidal categories, Abhishek Banerjee

The idea of doing algebraic geometry over a symmetric monoidal category \( (C, \otimes, 1) \) may be found in Deligne’s famous work *Catégories tannakiennes*. In particular, when the category \( C \) is taken to be \( k - \text{Mod} \), the category of modules over a commutative ring \( k \), we obtain the usual geometry of schemes over \( k \).

In this lecture, I will present a systematic development of the basics of algebraic geometry for schemes over a symmetric monoidal category \( (C, \otimes, 1) \). The precise notion of a scheme over \( (C, \otimes, 1) \) that we shall use is due to Toën and Vaquié.

In particular, I will present my work with schemes over \( (C, \otimes, 1) \) in the following contexts along with discussing avenues for further research:

1. We introduce quasi-coherent sheaves for schemes over \( (C, \otimes, 1) \), along with study of Picard groups, as also the sheaf properties of the relative Picard functor in this context.

2. We develop an appropriate notion for “Noetherian schemes” in the relative geometry over \( (C, \otimes, 1) \). We also develop notions of integral schemes, irreducible and reduced schemes over \( (C, \otimes, 1) \), along with giving meaning to the idea of residue fields and local rings of points in this context.

3. One of our major efforts is to find the correct notion for a “field object” in a symmetric monoidal category, i.e., a monoid object in \( (C, \otimes, 1) \) whose category of modules has properties similar to that of vector spaces over a field. In particular, we show a one-one correspondence between certain kinds of tensor functors between categories of quasi-coherent sheaves and points of a scheme over such a “field object.”

4. We study torsion theories in the category of quasi-coherent sheaves for schemes over \( (C, \otimes, 1) \). In particular, we give conditions under which the quasi-coherator of a sheaf of modules becomes a subsheaf. This has applications to the study of locally torsion free sheaves on schemes over \( (C, \otimes, 1) \).
We develop the theory of the derived category of quasi-coherent sheaves for schemes over \((\mathbb{C}, \otimes, 1)\). We feel that this is particularly interesting since we do not impose any Noetherian condition on the schemes.

We extend the framework of algebraic geometry over \((\mathbb{C}, \otimes, 1)\) to group schemes. In particular, we extend a classical result of Deligne on the annihilation of group schemes of finite rank to the case of schemes over \((\mathbb{C}, \otimes, 1)\).

Center of generalized quantum groups via Weyl groupoids, Punita Batra and Hiroyuki Yamane

Center of generalized quantum groups via Weyl groupoids

Punita Batra (Harish-Chandra Research Institute, India)
Hiroyuki Yamane (University of Toyama, Japan)

Abstract:
Let \(V\) be a finite-rank free \(\mathbb{Z}\)-module. Let \(\mathcal{P}(V)\) be the power set of \(V\). Let \(\mathcal{B}(V)\) be a set of all \(\mathbb{Z}\)-bases of \(V\). Let \(R \in \mathcal{P}(V) \setminus \{\emptyset\}\). For \(B \in \mathcal{B}(V)\), let \(R^{B,+} := (\bigoplus_{b \in B} \mathbb{Z}_{\geq 0} b) \cap R\) and \(R^{B,-} := -R^{B,+}\). We say that \(B \in \mathcal{B}(V)\) is a base of \(R\) if (i) \(R = R^{B,+} \cup R^{B,-}\) and (ii) \(\forall \alpha \in B, \mathbb{Z}\alpha \cap R = \{\alpha, -\alpha\}\). Let \(\mathcal{B}(R)\) be a set of all bases of \(R\). Let \(\mathbb{B}\) be a non-empty subset of \(\mathcal{B}(R)\). Assume that \(\forall B \in \mathbb{B}, \forall \alpha \in B, \exists \tau_\alpha(B) \in \mathbb{B}, R^{\tau_\alpha(B),+} \cap R^{B,-} = \{-\alpha\}\). If \(R\) is a finite set, \(\mathbb{B} = \mathcal{B}(R)\). Let \(W\) be the Weyl groupoid defined for \((R, \mathbb{B})\). If \(R\) is the root system of a semisimple complex Lie algebra \(\mathfrak{g}\), \(W\) is virtually the same as the Weyl group of \(\mathfrak{g}\). Let \(U(\chi)\) be a generalized quantum group, which is defined as the Drinfeld double of a Nichols algebra of diagonal-type. Let \(R(\chi)\) be the Kharchenko’s root system defined for \(U(\chi)\). Assume that \(R(\chi)\) is a finite set and \(\chi(\alpha, \alpha) \neq 1\) for all \(\alpha \in R(\chi)\). Let \(\mathcal{Z}(\chi)\) be the center of \(U(\chi)\). Let \(\mathcal{H}: \mathcal{Z}(\chi) \to U^0(\chi)\) be the Harish-Chandra map, which is injective. We studied \(\mathcal{H}\) using \(W\). In particular, we have obtained a concrete formula characterizing the image of \(\mathcal{H}\).

Lipschitz conditions in Laguerre hypergoup, Selma Negzaoui

Lipschitz conditions in Laguerre hypergoup, Selma Negzaoui University of Tu-
In this talk we show analogous of Titchmarsh’s theorems for the Laguerre transform. More precisely, we give a Lipschitz type condition on $f$ in $L^p(\mathbb{K})$ for which its Laguerre transform belongs to $L^\beta(\check{\mathbb{K}})$ for some values of $\beta$, where $\mathbb{K} = [0, +\infty) \times \mathbb{R}$ and $\check{\mathbb{K}}$ is its dual. In the particular case, when $p = 2$, we provide equivalence theorem : we get a characterization of the space $Lip_\alpha(\gamma, 2)$ of Lipschitz class functions by means of asymptotic estimate growth of the norm of their Laguerre transform for $0 < \gamma < 1$. Furthermore, we introduce Laguerre-Dini-Lipschitz class $LDLip_\alpha(\gamma, \delta, p)$ and we obtain analogous of Titchmarsh’s theorems in this occurrence.

Self-similar measures: asymptotic bounds for the dimension and Fourier decay of smooth images, Carolina A. Mosquera and Pablo Shmerkin

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R. Kaufman and M. Tsujii proved that the Fourier transform of self-similar measures has a power decay outside of a sparse set of frequencies. We present a version of this result for homogeneous self-similar measures, with quantitative estimates, and derive several applications: (1) non-linear smooth images of homogeneous self-similar measures have a power Fourier decay, (2) convolving with a homogeneous self-similar measure increases correlation dimension by a quantitative amount, (3) the dimension and Frostman exponent of (biased) Bernoulli convolutions tend to 1 as the contraction ratio tends to 1, at an explicit quantitative rate. The results are based on a joint work with Pablo Shmerkin.

Representation varieties of orbifold fundamental groups, Florent Schaffhauser

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Let us denote by $\mathbb{H}_\mathbb{R}^2$ the real hyperbolic plane and let $\Gamma$ be a discrete subgroup of $\text{PGL}(2; \mathbb{R})$, the full isometry group of $\mathbb{H}_\mathbb{R}^2$. The quotient space $Y := [\Gamma \backslash \mathbb{H}_\mathbb{R}^2]$ admits a canonical structure of dianalytic orbifold of dimension 2 (or orbifold
Klein surface), whose orbifold fundamental group can be identified with $\Gamma$. We study representations of $\Gamma$ into a real Lie group $G$, for instance the extended unitary group $U(n) \rtimes \mathbb{Z}/2\mathbb{Z}$ or the group of projective transformations $\text{PGL}(n; \mathbb{R})$, in terms of geometric objects on $Y$. When $Y$ is compact, we obtain representation spaces $\text{Hom}(\Gamma, G)/G$ whose topology can sometimes be studied explicitly. For $G = U(n) \rtimes \mathbb{Z}/2\mathbb{Z}$, this was achieved in earlier joint work with Chiu-Chu Melissa Liu, while for $G = \text{PGL}(n; \mathbb{R})$, we show, in a recent collaboration with Daniele Alessandrini and Gye-Seon Lee, that there is a notion of Hitchin component in the representation space of $\Gamma$, homeomorphic to an open ball of computable dimension, depending only on the topology of $Y$. Our formula generalizes known results due to Thurston and Choi-Goldman, corresponding respectively to the cases $n = 2$ and $n = 3$. As an application, we obtain a classification of all discrete subgroups of $\text{PGL}(2; \mathbb{R})$ (in particular, hyperbolic Coxeter groups) that admit 0 or 1-dimensional Hitchin components. Our result also shows that Hitchin’s formula for the dimension of Hitchin components of fundamental groups of closed orientable surfaces generalizes to orientable surfaces with boundary, for which the notion of Hitchin component was already worked out by Labourie and McShane for topological fundamental groups and in fact coincides with ours. We also deduce from our techniques that Hitchin representations of $\Gamma$ into $\text{PGL}(n; \mathbb{R})$ are discrete and faithful, thus generalizing results obtained by Labourie in the torsion-free case.

**Spectral decomposition of dichotomous linear operators**, Monika Winklmeier
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Let $S$ be a densely defined linear operator on a Banach space $X$. If its resolvent is uniformly bounded along the imaginary axis, then there exist $S$-invariant spectral subspaces $X_\pm$ such that $X = X_+ \oplus X_-$ and the spectrum of the restrictions $S|_{X_\pm}$ belongs either to left or to the right complex half plane. The equality $X = X_+ \oplus X_-$ holds if a certain pair of projections is bounded.

This talk is based on a joint work with Christian Wyss.

**Geodesic properties of the differential process generated by complex interpo-**
Interpolation theory is a classical field of functional analysis, related to operator theory, non-linear theory and homological theory of Banach spaces. An interpolation method associates to a pair of Banach spaces embedded in a common linear space, a continuous segment of intermediate spaces, with the following Riesz-Thorin property: an operator which is bounded on both the initial spaces must be bounded as an operator on each intermediate space.

In 1984, Rochberg and Weiss define a differential process for interpolation scales, inducing exact sequences (or twisted sums) of each intermediate space, relating interpolation to homological theory. In 1995, Daher observes that complex interpolation scales of uniformly convex spaces induce uniform homeomorphisms between the unit spheres on the scale, relating interpolation to non-linear theory.

We investigate here the relation between these two aspects of interpolation, characterizing properties of the differential process in one single point by the shape of the scale of interpolation, and in particular by the geodesic relation between the spaces of the boundary.

Stability results are obtained for the bounded, splitting or singular character of the differential process associated to complex interpolation of a pair (or in some instances a family) of Banach spaces. The problems studied here are whether boundedness (resp. splitting, singularity) at some interior point implies the same in a neighborhood of the point (local stability), on the whole domain (global stability), or is equivalent to some conditions on the spaces of the boundary (scale stability). It is proved that there is global and scale stability for splitting, for couples of uniformly convex Köthe spaces; but there is no stability for families of Köthe spaces. Several scale stability results are also obtained for interpolation of couples of uniformly convex spaces with common monotone basis.

Spectral Geometry on Stratified Spaces, Luiz Hartmann Departamento de Matemática - Universidade Federal de São Carlos. Rod. Washington Luís, Km 235, 13565-905, São Carlos SP, Brazil. hartmann@dm.ufscar.br

In his seminal work “Spectral Geometry of Singular Riemannian Spaces”, Jeff
Cheeger started an influential program on spectral analysis on stratified spaces with singular Riemannian metrics. The ultimate goal of the program is to establish resolvent trace asymptotics in this very general setting. We propose to discuss the development of Cheeger’s program until the recent days.

**Ramsey methods and indiscernibles in nonseparable Banach spaces,** Christina Brech, Jordi Lopez-Abad and Stevo Todorcevic brech@ime.usp.br

In the present work we present combinatorial structures on uncountable index sets needed to understand the complexity of sets of indiscernibles in nonseparable Banach spaces. Given a structure, a set of indiscernibles is a subset of the universe, with a total ordering, such that any two increasing tuples of the same length have the same properties. In the context of Banach spaces, the natural notion of indiscernibility is that of subsymmetric sequences. Tsirelson’s construction of a Banach space with no copies of $c_0$ and $\ell_p$ gives an example of an infinite dimensional reflexive Banach space where only finite indiscernible sets exist.

In order to get various similar objects in the nonseparable setting, we generalize, to uncountable index sets, the combinatorial structures used in Tsirelson’s construction, such as products of generalized Schreier families. Lopez-Abad and Todorcevic had previously obtained generalizations of the Schreier family to larger index sets, suggesting that this could be done for index sets of some cardinalities. In our work, we present a method to construct such generalizations and, moreover, to operate with them, on index sets with cardinality $\kappa$ below the first Mahlo cardinal, and thus getting a version of Tsirelson space of density $\kappa$ for each such $\kappa$.

The method involves an inductive argument where trees are used to pass from lower to higher cardinalities, by introducing the operation $\odot$ which outputs families of (arbitrary) finite sets in the whole tree out of families of finite sets formed exclusively by chains or exclusively by (particular) antichains. The main difficulty is to prove that the operation $\odot$ preserves products and this requires a deep combinatorial analysis of sequences of finitely generated subtrees, using Ramsey methods.
Positivity results for indefinite sublinear problems, Uriel Kaufmann, Humberto Ramos Quoirin and Kenichiro Umezu  urielkaufmann@gmail.com

Let $\Omega \subset \mathbb{R}^N$, $N \geq 1$, be a smooth bounded domain and let $a \in L^r(\Omega)$, $r > N$, be a possibly sign changing function with $a^+ \neq 0$. We consider diverse issues related with the existence of strictly positive solutions for nonlinear elliptic problems of the form

$$(P_{a,q}) \quad \begin{cases} -\Delta u = a(x)u^q & \text{in } \Omega, \\ u = 0 & \text{on } \partial \Omega, \end{cases}$$

where $0 < q < 1$. Let us note that neither the strong maximum principle nor Hopf’s lemma can be applied to $(P_{a,q})$, which makes the existence of positive solutions a nontrivial matter.

On The Generalized Matsumoto Relation, Elif Dalyan elifylmz@yahoo.com

There is an interaction between mapping classes and structures in low dimensional topology. Indeed Y. Matsumoto discovered a relation with eight right Dehn twists in Mapping Class Group of genus-2 surface which defines a genus-2 Lefschetz fibration over 2-sphere. Then M. Korkmaz generalized that Matsumoto relation to any genus $g > 1$ surface which defines a genus-$g > 1$ Lefschetz fibration over 2-sphere. In order to classify these 4-manifolds, understand its invariants we can work with mapping class group elements, their monodromies. In this work we study this generalized Matsumoto Relation, we give an alternative proof of this relation, in which we used only the well known relations in Mapping Class Groups and obtain a different set of curves from Korkmaz and Matsumoto.

Mathematics school textbook design and development framework, Fabio Simas and Augusto Teixeira

In this talk, we will overview some of the recent research that addresses the importance of the textbook on teaching, learning and on bringing some of the research in Mathematics Education to the school and teachers. For example, recent
research has shown that textbooks influence teachers’ strategies in classrooms, a quantitative study with thousands of students, concluded an important correlation between the textbook’s opportunity to learn, in terms of demanded cognition by the tasks presented in the book, and the student’s scores in a standardized exam.

Given the relevance of the subject, we will turn to some of the factors that influence the textbook design, its development framework and the overall quality of the books. These factors range from their source of financing (public or private), the type of institution behind their development (profit or non-profit organizations) and the license of the resulting material. The impact on teaching and learning as well as the acceptance by teachers of the final outcome can be exemplified by various successful initiatives such as the one by the National Science Foundation on the 1990’s to develop series of textbooks in Mathematics for the USA. As well as the Sesamath Project, on the 2000’s that developed an open source series of Mathematics textbooks by an association of teachers, among others.

We believe that a cost/benefit analysis of all these decisions should guide us through the process of choosing a methodology of development that most positively impacts our classrooms. In this spirit, I will present a framework and an initiative to build an open source collection of Mathematics textbooks for high school that addresses all those issues.

**Spatial and Spatio-Temporal Nonlinear Processes**, Wolfgang Schmid, Philipp Otto and Robert Garthoff  schmid@europa-uni.de

In this talk we present a new spatial model that incorporates heteroscedastic variances depending on neighboring locations. The proposed process is regarded as the spatial equivalent of the temporal autoregressive conditional heteroscedastic (ARCH) model. We show as well how the introduced spatial ARCH model can be used in spatiotemporal settings. The process turns out to be strictly and weakly stationary under some conditions on the noise process and the weight matrix. Although it possesses several properties of a temporal ARCH process some important features are no longer fulfilled. The conditional variance of the process depends on all locations and only for an oriented process it is a function of the locations lying closer to the center. Moreover, the squared process is not a spatial autoregressive process. In order to estimate the parameters of the spatial
process the maximum-likelihood approach is applied. For a certain type of weight matrices it is proved that the estimators are asymptotic normally distributed. Via Monte Carlo simulations, we illustrate the performance of the estimator for various spatial weighting matrices and for a finite sample size. Moreover, we combine the known spatial autoregressive model with the spatial ARCH model assuming heteroscedastic errors. Eventually, the proposed autoregressive process is illustrated using an empirical example. Specifically, we model lung cancer mortality in 3108 U.S. counties and compare the introduced model with two benchmark approaches.

The Use of Videos To Learn, Teach and Popularize Mathematics and Statistics: The CineClub Project of The Fluminense Federal University, Humberto J. Bortolossi Fluminense Federal University – Niterói – Brazil hjbortol@gmail.com

In the last decade there has been a notable increase in the production of audiovisual materials (documentaries, animations, films, short films) related to Mathematics and Statistics: BBC and NOVA documentaries, “Isto é Matemática” (presented by Rogério Martins) and “Numberphile” YouTube channels, TED-Ed videos, “Dimensions” and “CHAOS” short videos idealized and produced by Étienne Ghys et al., some “The Simpsons” episodes, to name just a few. With the objective of cataloging these materials and maximizing the didactic use of this type of resource beyond its simple screening, a team of lecturers, graduate and undergraduate students of the Federal Fluminense University (Brazil) has been working on the creation and maintenance of a blog with detailed guidelines for the use of the various videos in the educational context and, also, for scientific popularization to the general public. The guidelines were tested in schools and events of public awareness of Mathematics and Statistics in Brazil, having reached more than 2300 people in 2017. Each videos’ guideline contains an information data sheet (with production date, length, type, voice and subtitle languages, license of use, etc.), indications of objectives that can be achieved in the basic school context, suggestions of questions and activities to be developed after the exhibition, as well as a compilation of supporting references pointing to diverse connections in different subjects with the content presented in the video. All work is developed
with support in the Theory of Narratives, in their educational, philosophical and neuroscientific aspects.

On representation and character varieties of some one-relator groups, Alexandra Admiralova al.admiralova@gmail.com

Let $G = \langle x_1, x_2, \ldots, x_g \rangle$ be a finitely generated group and $K$ be an algebraically closed field with $\text{char } K = 0$. The set $\text{Hom}(G, GL_n(K))$ has a natural structure of an affine $K$-variety and is denoted by $R_n(G)$. It is called the representation variety of the group $G$. The group $GL_n(K)$ acts on $R_n(G)$ by conjugation and the corresponding category factor $R_n(G) // G$ is denoted by $X_n(G)$ and called the character variety of the group $G$.

We consider representation varieties $R_n(G_i)$ and character varieties $X_n(G_i)$ of one-relator groups with the following presentations:

$G_1 = \langle (x_1, y_1, \ldots, x_g, y_g) \in GL_n(k)^{2g} \mid t([x_1, y_1], \ldots, [x_g, y_g])^p t^{-1} = ([x_1, y_1], \ldots, [x_g, y_g])^q \rangle$

$G_2 = \langle (x_1, x_2, \ldots, x_g) \in GL_n(k)^{g} \mid t(x_1^2 x_2^2 \ldots x_g^2)^p t^{-1} = (x_1^2 x_2^2 \ldots x_g^2)^q \rangle$

$G_3 = \langle (x_1, \ldots, x_s, y_1, \ldots, y_g, z_1, \ldots, z_g) \in GL_n(k)^{2g+s} \mid t([y_1, z_1] \ldots [y_g, z_g]W(x_1, \ldots, x_s))^p t^{-1} = ([y_1, z_1] \ldots [y_g, z_g]W(x_1, \ldots, x_s))^q \rangle$

In all the relations above we assume that $p$ and $q$ are integers such that $p > |q| \geq 1$ and $W(x_1, \ldots, x_s)$ is an arbitrary cyclically reduced word on generators $x_1, \ldots, x_s$.

For groups $G_i$ we investigate the structure of varieties $R_n(G_i)$ and $X_n(G_i)$, describe their irreducible components and calculate their dimensions. We also prove that representation varieties $R_n(G_i)$ are rational.

Mathematical modeling of fluid flow inside a knee joint using squeeze-film approach, Raja Sekhar P. Gangavamsam and Timir Karmakar rajas@iitkgp.ac.in

Modeling fluid flow inside a knee-joint is very essential to estimate various physical insights depending on the internal characteristics of the knee. The literature in this direction indicates that the articular cartilage can be treated as a rigid
porous medium and the synovial fluid as a Newtonian fluid in nature where the interface of the articular cartilage and the synovial fluid are flat. Knee joint is where the two largest bones of the human body tibia and femur meet. The most important aim of such studies is to estimate the contact time for femoral condyles to reach the tibial plateau. The permeability of the articular cartilage depends on the proteoglycan matrix, in particular glycosaminoglycan (GAG) chains and the arrangement of the collagen fibres which are responsible for the anisotropic nature of the cartilage. Therefore, we believe that approximating cartilage as anisotropic porous matter would be a more realistic approximation. Hence, we consider a theoretical model of squeeze-film flow in the presence of a thin porous bed. It is assumed that a flat bearing is approaching towards the porous bed. The gap between the porous bed and the bearing is assumed to be filled with a Newtonian fluid. Assuming that the fluid is Newtonian, we use Navier-Stokes equation in the fluid region and Darcy equation in the fluid saturated anisotropic porous region. Lubrication approximation is used to the hydrodynamic equation of motion in the gap and in the porous region. In case of constant load, we have estimated the time duration a healthy human knee remains fluid lubricated. While in case of a constant velocity, we have estimated the load a human knee can sustain. We have used a regular asymptotic expansion in terms of Darcy number and estimated the corresponding contact time.

Some Results On Universal Function Series, Shakro Tetunashvili

In our talk various questions related to universal function series are discussed. Among mentioned ones are: conditions for the existence of universal series with respect to a system of measurable and almost everywhere finite functions, a generalization of Menshov’s theorem on universal trigonometric series, different properties of coefficients of universal function series.

Parikh Rewriting Systems—A New Approach To The Injectivity Problem Of Parikh Matrices, Wen Chean Teh

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Some Results On Universal Function Series, Shakro Tetunashvili stetun@rmi.ge

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Parikh Rewriting Systems—A New Approach To The Injectivity Problem Of Parikh Matrices, Wen Chean Teh Universiti Sains Malaysia, Malaysia dasmenteh@usm.my
The classical Parikh’s theorem states that the set of Parikh vectors of a context-free language is semilinear. The Parikh matrix mapping is a canonical extension of the Parikh mapping. The Parikh matrix of a word contains some of its binomial coefficients depending on the underlying ordered alphabet. Hence, Parikh matrices is a powerful tool to study subword occurrences. Two words are $M$-equivalent iff they share the same Parikh matrix. The characterization of $M$-equivalence, known as the injectivity problem, belongs to the classical word inference problem, which studies reconstruction of a word from some of its binomial coefficients. Although being the most studied problem in Parikh matrices, it remains elusive even for the ternary alphabet.

In [Theoret. Comput. Sci. 411 (2010) 1818–1827; MR2650344] Salomaa provided some solution for the ternary alphabet in terms of a Thue system. Nevertheless, the associated rewriting rules do not preserve $M$-equivalence. To compensate, a counter is embedded into Salomaa’s system to keep track of the unique binomial coefficient that decides whether $M$-equivalence is preserved after finitely many applications of the rewriting rules. Later Atanasiu proposed a wider class of $M$-equivalence preserving rewriting rules for the ternary alphabet, based on Salomaa’s system.

Motivated by Salomaa’s and Atanasiu’s work, we propose the notion of Parikh rewriting system as a generalization of Salomaa’s system. Every Parikh rewriting system induces a Thue system consisting of irreducible transformations that completely characterizes $M$-equivalence. The irreducible transformations for a simple and canonical Parikh rewriting system coincide with the standard elementary rewriting rules that characterize $M$-equivalence for the binary alphabet. Therefore, it is conceivable that an ingenious Parikh rewriting system with decidable rewriting rules for the ternary alphabet can be found such that all its irreducible transformations can be effectively characterized and thus finally conquering the injectivity problem.

**Time distributed-order wave equation on infinite domain**, Sanja Konjik *University of Novi Sad, Serbia* sanja.konjik@dmi.uns.ac.rs
Differential and integral operators of noninteger order have become an indispensable tool for modeling certain phenomena in physics, mechanics, engineering, economics, medicine, etc. Over the past years there has been an extensive study of wave propagation in viscoelastic materials within the fractional framework. In this talk we discuss various possibilities for generalizations of the classical wave equation by the use of fractional derivatives of constant real order. As a novel approach, we propose distributed order fractional model to describe wave propagation in viscoelastic infinite media, and study existence and uniqueness of fundamental solutions for the corresponding generalized Cauchy problem. Some particular cases of distributed order fractional constitutive stress-strain relations are examined in details, and numerical experiments are presented to illustrate theoretical results.

The talk is based on collaborations with Lj. Oparnica and D. Zorica.

Distribution of prime geodesics on Riemann surfaces and hyperbolic 3–manifolds, Muharem Avdispahić

Let $\mathcal{H}$ denote the upper half-plane, resp. half-space, equipped with the hyperbolic metric, $\Gamma$ be a Fuchsian, resp. Kleinian, group and $\pi_\Gamma (x)$ be the number of prime geodesics $P$ on the manifold $\Gamma \backslash \mathcal{H}$ with the length $l(P) \leq \log x$. We prove

**Theorem:** Let $\Gamma \subset PSL(2, \mathbb{R})$ be a cocompact or a noncompact cofinite Fuchsian group satisfying the condition $\sum_{\delta > 0} \frac{x^{\beta-1/2}}{\delta^2} = O\left(\frac{1}{1+(\log x)^2}\right)$ ($x \to \infty$),

where $\beta + i \delta$ are poles of the scattering determinant. For $\varepsilon > 0$ arbitrarily small, there exists a set $E$ of finite logarithmic measure $\mu^x E = \int_E \frac{d\gamma}{\gamma}$ such that

$$\pi_\Gamma (x) = li (x) + \sum_{\frac{\gamma}{10} < s_n < 1} li (x^{s_n}) + O \left( x^{\frac{7}{10}} (\log x)^{-\frac{1}{2}} (\log \log x)^{\frac{1}{3} + \varepsilon} \right) (x \to \infty, x \notin E).$$

Here $s_n$ denote (finitely many) real zeros of the Selberg zeta function $Z_\Gamma$ in the critical strip.

**Theorem:** For the modular group $\Gamma = PSL(2, \mathbb{Z})$, and $\varepsilon > 0$ arbitrarily small, one has

$$\pi_\Gamma (x) = li (x) + O \left( x^{\frac{2}{3}} (\log \log x)^{\frac{1}{3} + \varepsilon} \right) (x \to \infty)$$
outside a set $F$ such that $\mu^x F < \infty$.

**Theorem:** Let $\Gamma \subset PSL(2, \mathbb{C})$ be a cocompact Kleinian group or a noncompact congruence group for some imaginary quadratic number field. Then there exists a set $G, \mu^x G < \infty$, such that

$$\pi_T(x) = li(x^2) + \sum_{\frac{13}{9} < s_n < 2} li(x^{s_n}) + O \left( x^{\frac{13}{9}} (\log x)^{-\frac{2}{3}} (\log\log x)^{\frac{2}{3}+\varepsilon} \right) (x \to \infty, x \notin G).$$

**Remark:** The above exponents $\frac{7}{10}, \frac{2}{3}$ and $\frac{13}{9}$ in the error term of the Gallagher-type prime geodesic theorem are to be compared to unconditional estimates $\frac{3}{4}, \frac{25}{36}$ and $\frac{5}{3}$ due to Randol, Soundararajan-Young and Sarnak, resp.

**Impulse effect in a food-limited population model**, Fatma Karakoç *Ankara University, Turkey fkarakoc@ankara.edu.tr*

In this talk, we present impulse effect in a food-limited population model with piecewise constant argument. We investigate asymptotic behaviour of the positive equilibrium point of the model. First, we introduce the relation between the solutions of an impulsive differential equation with piecewise constant argument and difference equations. Then sufficient conditions for the oscillation of the positive equilibrium point are obtained. Linearized oscillation theory for difference equations is the main tool to prove the results. Moreover, it is proved that every non-oscillatory solution approaches to positive equilibrium point as $t$ tends to infinity. Finally, it is showed that impulse effect can change the asymptotic behaviour of the solutions.

**On the projection of self-adjoint operators**, Maria L. Arias and Maria C. Gonzalez *ariasmlau@gmail.com*

Let $\mathcal{H}$ be a Hilbert space and $L(\mathcal{H})$ be the algebra of bounded linear operators on $\mathcal{H}$. Given $A, B$ two subsets of $L(\mathcal{H})$ we denote by $AB = \{T \in L(\mathcal{H}) : T = AB \text{ for some } A \in A \text{ and } B \in B\}$. During the last years, we studied this set for different classes $A$ and $B$ of operators in $L(\mathcal{H})$. For example, we considered the classes of unitary operators, partial isometries, orthogonal and oblique projections,
among others. More precisely, we described $AB$ and, given $T \in AB$, we studied the projection sets $\{A \in A : T = AB \text{ for some } B \in B\}$ and $\{B \in B : T = AB \text{ for some } A \in A\}$. Furthermore, when it was possible, we determined optimal factors in these last two sets. For instance, for the set $\mathcal{P}\mathcal{L}^+$, where $\mathcal{P}$ denotes the set of orthogonal projections and $\mathcal{L}^+$ the cone of semi-definite positive operators, we obtained optimal factors considering the usual order for self-adjoint operators (i.e., $A \leq B$ if $B - A \in \mathcal{L}^+$) for all $T \in \mathcal{P}\mathcal{L}^+$. Our objective now is to study the bigger set $\mathcal{P}\mathcal{L}^h$ where $\mathcal{L}^h$ denotes the set of self-adjoint operators. We compare this set with $\mathcal{P}\mathcal{L}^+$ and, again, we describe this set and characterize the factors in $\mathcal{P}$ and $\mathcal{L}^h$ for a given $T \in \mathcal{P}\mathcal{L}^h$. Concerning the optimality of factors in $\mathcal{P}$ and $\mathcal{L}^h$, we get results considering different criteria with respect, not only to the usual order between selfadjoint operators, but also to the minus order of operators and to the operator norm. Our goal in this presentation is to survey all these results concerning the set $\mathcal{P}\mathcal{L}^h$. This is a joint work with M. Celeste Gonzalez.

On the solvability of nonhomogeneous boundary value problem for the Burgers equation in the angular domain, Muvasharkhan Jenaliyev, Meiramkul Amangaliyeva and Murat Ramazanov

In the domain $G = \{x, t : 0 < x < k t, t > 0\}$ we consider the boundary value problem for Burgers equation:

$$
\begin{align*}
    \left\{ \begin{array}{l}
    w_t + b w w_x - a^2 w_{xx} = 0, \quad \{x, t\} \in G, \\
    w\big|_{x=0} = w_0(t), \quad w\big|_{x=kt} = w_1(t),
    \end{array} \right.
\end{align*}
$$

(1)

where $w_0(t), w_1(t), b > 0, k > 0$ are some given on $(0, \infty)$ functions and real numbers.

The nonhomogeneous boundary problem (1) is reduced to the problem on the solvability for the system of Volterra integral equations of second kind.

In this report we consider various special cases of Volterra integral equations and prove some lemmas which establish properties of integral operators in weighted space of essentially bounded functions. We prove the existence and properties of non-trivial solutions to the system of homogeneous integral equations. On the basis of lemmas the solvability theorems of the nonhomogeneous
boundary value problem for the Burgers equation in infinite angular domain are established.


**Multilayered flows in the shallow water limit: dynamics and loss of hyperbolicity in a mixed type PDE**, Francisco de Melo Viríssimo and Paul A. Milewski f.de.melo.virissimo@bath.ac.uk

In this presentation, we will formulate and discuss the problem of density stratified interfacial flows in the shallow-water limit. This type of flow occurs in nature with the atmosphere and ocean as prime examples.

Mathematical studies of these are particularly important, since wave motion tends not to be resolved by most numerical climate models due to their fast scales, and thus need to be understood and parameterized. For example waves may break and dissipate energy or mix the underlying fluids and affect the medium in which they are propagating. Consequently this research will both increase the understanding of internal waves, and have an impact on future climate models.

We will focus our attention on the two and three-layer flows, without the so-called Boussinesq approximation which requires small density differences. This is a simplified model for geophysical situations, but it is not too simplified: the model has both barotropic (fast waves affecting the whole fluid uniformly) and baroclinic modes (slower waves with more internal structure).

The governing equations will be derived and the dynamics of their solutions will be studied from both analytical and numerical points of view, particularly the
issue of whether the solutions maintain hyperbolicity (i.e. wave-like behaviour). Explicity criteria for transition to the elliptic regime will be provided using new dynamical system techniques. The existence of invariant hyperbolic regions will be proven and examples will be constructed using the so-called simple waves. These invariant regions are very important as they guarantee the well-posedness of the problem (in the sense of J. Hadamard). In addition, some of the techniques presented can be also applied to more general mixed-type systems of PDEs. Extensions and future work will be presented at the end.

**Causal detectability for linear descriptor systems**, Mahendra K. Gupta, Nutan K. Tomar and Raghunathan Rengaswamy mahendra14389@gmail.com

Consider the linear descriptor systems of the form

\[
\begin{align*}
E \dot{x} &= Ax + Bu, \quad (2a) \\
y &= Cx, \quad (2b)
\end{align*}
\]

where \( x \in \mathbb{R}^n \), \( u \in \mathbb{R}^k \), \( y \in \mathbb{R}^p \) are the state vector, the input vector, and the output vector, respectively. \( E, A \in \mathbb{R}^{m \times n}, B \in \mathbb{R}^{m \times k}, C \in \mathbb{R}^{p \times n} \) are known constant matrices. During past few decades, a lot of work has been done on various types of observer design for the systems of the form (2), and the references therein. Among all the observers, Luenberger observers were paid the most attention due to its explicit nature. Several techniques have been developed to design Luenberger observer for the descriptor system (2) and sufficient conditions on system operators have been provided for the existence of the Luenberger observer. Hou and Müller have proved that a rectangular descriptor system (2) can be observed by a Luenberger observer if and only if it is causally detectable. But these authors have given the condition of causal detectability of the system on a transformed system that can only be obtained by applying a finite number of orthogonal transformations on the original system. Thus without getting the transformed system, it is not possible to know that for a given descriptor system a Luenberger observer can be designed or not. In this work, the causal observability has been established in terms of system coefficient matrices. Therefore, necessary and sufficient conditions for the existence of Luenberger observers are provided in terms of system matrices.
Congruences for matrix permanents, Ian Wanless ian.wanless@monash.edu

The permanent is a matrix function (related to the determinant) that is useful in many counting problems. We consider permanents of integer matrices in classes of combinatorial interest, and ask whether there are any non-trivial congruences satisfied by those permanents. A sample result, phrased in graph theoretic terms, is this:

Theorem. Let $G$ be a $k$-regular bipartite graph with $n$ vertices in each partite set. If $n$ is odd and $k \equiv 0 \mod 4$ then the number of perfect matchings in $G$ is a multiple of 4.

As an application of the congruences we have discovered, we have been able to show that in Latin squares of order $2 \mod 4$ the number of transversals must be a multiple of 4. It had previously been shown by Balasubramanian that this number was even.

Star-critical Ramsey numbers of forests versus complete graphs, Ghaffar Raeisi and Azam Kamranian g.raeisi@math.iut.ac.ir

For given graphs $G, G_1, G_2$, we write $G \rightarrow (G_1, G_2)$ if in any red-blue coloring of the edges of $G$, there is either a red copy of $G_1$ or a blue copy of $G_2$. For given graphs $G_1$ and $G_2$, the Ramsey number $R(G_1, G_2)$ is defined as the smallest positive integer $n$ such that $K_n \rightarrow (G_1, G_2)$. The existence of such a positive integer is guaranteed by the Ramsey’s classical result.

To investigate the Ramsey properties of subgraphs of $K_{R(G_1, G_2)}$, several other definitions were introduced. In 1978, Erdős and Faudree considered the restricted size Ramsey number $R^*(G_1, G_2)$, defined as the minimum number of edges in a subgraph $H$ of $K_{R(G_1, G_2)}$ such that $H \rightarrow (G_1, G_2)$. In 2011, Hook and Isaak introduced the definition of the star-critical Ramsey number $R_*(G_1, G_2)$ which is defined as the

$$R_*(G_1, G_2) = \min\{\delta(H) : H \subseteq K_{R(G_1, G_2)} \text{ and } H \rightarrow (G_1, G_2)\}.$$

It is easy to see that $R^*(G_1, G_2) \leq \binom{R(G_1, G_2) - 1}{2} + R_*(G_1, G_2)$. In this talk, the exact value of the star-critical Ramsey number of any forest versus any complete...
graph will be computed exactly; Let $F$ be an arbitrary forest and $n(F)$ denote the number of vertices of the largest component of $F$. Then, for every $m \geq 3$,

$$R_*(F, K_m) = \max_{1 \leq j \leq n(F)} \{ (j - 1)(m - 3) + \sum_{i=j}^{n(F)} i k_i(F) \},$$

where, $k_i(F)$ is number of components of $F$ with exactly $i$ vertices.

**Hermitian geometry of real flag manifolds**, Viviana del Barco

The aim of this talk is to present recent results about the Hermitian geometry of real flag manifolds $M = G/P_{\Theta}$. Here $G$ is a split form of a simple complex Lie group and $P_{\Theta}$ is the parabolic subgroup associated to the subset of simple roots $\Theta$. The Hermitian structures considered are those invariant by the maximal compact subgroup $K$ of $G$, which acts transitively on $M$, thus giving the presentation $M = K/K \cap P_{\Theta}$.

I will address results regarding the classification of real flags admitting $K$-invariant almost complex structures. One can prove that these are never Kähler, since they cannot carry symplectic structures. Nevertheless, other of the sixteen types of Hermitian structures with respect to Gray-Hervella’s classification are obtained. For instance, integrable structures and non-integrable Hermitian structures of type $W_1 + W_3 + W_4$ appear on type $C$; these can be induced to the complex Stiefel manifold. These structures admit a unique metric connection with totally skew-symmetric torsion, according to Friedrich and Ivanov.

The main tool to achieve these results is the description of the isotropy representation of the real flags given by Patrão and San Martin. The talk is based on works (some on-going) in collaboration with Ana Paula Cruz de Freitas and Luiz San Martin.

**Algebraic aspects of finite cellular automata**, Alonso Castillo Ramirez
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Let $G$ be a group and $A$ a set. Denote by $A^G$ the set of all functions $x : G \to A$. A cellular automaton over $A^G$ is a transformation $\tau : A^G \to A^G$ that
is determined by a finite subset $S \subseteq G$, called a memory set of $\tau$, and a local function $\mu : A^S \to A$. The classical setting, which has been widely studied from a dynamical and computational perspective, occurs when $G = \mathbb{Z}^d$, $d \in \mathbb{N}$, and $A = \{0, 1\}$.

In joint work with Maximilien Gadouleau (Durham University), we have studied several algebraic aspects of cellular automata when $G$ and $A$ are both finite. In this case, the monoid $CA(G; A)$ consisting of all cellular automata over $A^G$ coincides with the endomorphism monoid of $A^G$ considered as a $G$-set with respect to the shift action of $G$ on $A^G$. Our main findings are the following:

1. We describe the structure of the group of units $ICA(G; A)$ of $CA(G; A)$ in terms of the structure of $G$ and the numbers $\alpha_{[H]}$ of $[H]$-periodic functions in $A^G$, where $[H]$ denotes the conjugacy class of $H \leq G$. The numbers $\alpha_{[H]}$ may be computed using the Möbius function of the subgroup lattice of $G$.

2. We show that $CA(G; A)$ cannot be generated by cellular automata with small memory sets: if $T$ generates $CA(G; A)$, then there exists $\tau \in T$ whose minimal memory set is $G$ itself.

3. We determine the minimal size of a generating set for $CA(\mathbb{Z}_{2^k} p; A)$ when $k \geq 0$, and $p$ is a prime or $p = 1$. We also provide upper and lower bounds for general finite cyclic groups.

4. We give a lower bound for the minimal size of a set $V \subseteq CA(G; A)$ such that $CA(G; A) = \langle ICA(G; A) \cup V \rangle$, and we show that this lower bound is achieved if and only if all the subgroups of $G$ are normal.

**Hopf bifurcation at infinity and dissipative vector fields of the plane**, Begoña Alarcón and Roland Rabanal
Universidade Federal Fluminense and Pontificia Universidad Católica del Perú balarcon@id.uff.br and rrabanal@pucp.edu.pe

We present one–parameter families of differentiable (not necessarily $C^1$) planar vector fields for which the infinity reverses its stability as the parameter goes through zero. These vector fields are defined on the complement of some compact ball centered at the origin and have isolated singularities. They may be considered as linear perturbations at infinity of a vector field with some spectral property, for instance, dissipativity. We also address the case concerning linear
perturbations of planar systems with a global period annulus. It is worth noting that the adopted approach is not restricted to consider vector fields which are strongly dominated by the linear part. Moreover, the Poincaré compactification is not applied in this work. Actually, we strongly focus on the change of the sign of an index defined at infinity.

This paper (DOI 10.1090/proc/13462) pretends to extend to the differentiable case a previous work published in DCDS-A by the first author (joint with C. Gutierrez and V. Guíñez). The authors of DCDS-A guarantee the change of stability at infinity of a one–parameter family of $C^1$–vector fields as the parameter goes through zero. This behaviour was presented as Hopf bifurcation at infinity.

**QHWM of the ”symplectic” and ”orthogonal” type Lie algebras of the matrix quantum pseudo differential operators**, Karina Batistelli and Carina Boyallian

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In this talk we will characterize the irreducible quasifinite highest weight modules of some subalgebras of the Lie algebra of matrix quantum pseudodifferential operators $N \times N$.

In order to do this, we will first give a complete description of the anti-involutions that preserve the principal gradation of the algebra of $N \times N$ matrix quantum pseudodifferential operators and we will describe the Lie subalgebras of its minus fixed points. We will obtain, up to conjugation, two families of anti-involutions that show quite different results when $n = N$ and $n < N$. We will then focus on the study of the “orthogonal” and “symplectic” type subalgebras found for case $n = N$, specifically the classification and realization of the quasifinite highest weight modules.

**The Weitzenböck Formula for the Fueter-Dirac Operator**, Andrés J. Moreno Ospina and Henrique Sá Earp *University of Campinas (Unicamp) amoreno0102@hotmail.com*
We find Weitzenböck formulae for the Fueter-Dirac operator which controls the infinitesimal deformations of an associative submanifold $Y^3$ in a 7–manifold $(M, \varphi)$ with a $G_2$–structure. Since, the tangent space at an associative submanifold $Y$ of the moduli space of associative deformation is identifying with the kernel of the Fueter-Dirac operator $\ker D \subset \nu(Y)$, we establish vanishing theorems under some positivity assumptions on curvature to conclude rigidity, i.e. that $Y$ has “essentially” no infinitesimal associative deformations, up to symmetries of $\varphi$, such that $Y$ is no fixed.

The general Weitzenböck formula is particularly mild in the nearly parallel case (i.e. $d\varphi = \tau_0 \ast_\varphi \varphi$ where $\tau_0 \neq 0$ is a constant) and in the locally conformal calibrated case (i.e. $d\varphi = 3\tau_1 \wedge \varphi$ where $\tau_1$ is a 1-form on $M$). As applications, we find a different proof of rigidity for the associative 3-sphere embedding in the round 7-sphere, which has a canonical nearly parallel $G_2$-structure, induced by the canonical Spin(7)-structure of $\mathbb{R}^8$. Finally, we construct an example of a rigid associative submanifold in a solvmanifold $S = \mathbb{R} \times_{\mu} N$, with a locally conformal calibrated $G_2$-structure $\varphi = \omega \wedge dr + \Omega_+$, where $N$ is a 6-dimensional nilpotent Lie group with a coupled SU(3)-structure $(\omega, \Omega_+)$, $\mu : \mathbb{R} \to \Aut(N)$ a Lie group representation and $\varphi = \omega \wedge dr + \Omega_+$ is a $G_2$-structure obtained from the left-invariant metric of the soluble Lie group $S$, the construction of this associative submanifold, naturally arise from the 3-dimensional Lie subalgebras $\mathfrak{p}$ of $\mathfrak{s} = \Lie(S)$ which are calibrated by $\varphi$, i.e. $\varphi|_\mathfrak{p} = \vol(\mathfrak{p})$.

**Automorphism groups of almost all finite 2-groups of coclass 2 are 2-groups**, Alireza Abdollahi and Nafiseh Rahmani alireza_abdollahi@yahoo.com

Using the coclass theory of finite 2-groups and thanks to ANUPQ package of GAP, we prove that there are only a finitely many finite 2-groups of coclass 2 whose automorphism groups are not 2-groups.

The research of the first author was in part supported by a grant (No. 96050219) from School of Mathematics, Institute for Research in Fundamental Sciences (IPM).
The main result of this paper is the following: if $F$ is any field and $R$ any $F$-subalgebra of the algebra $\mathbb{M}_n(F)$ of $n \times n$ matrices over $F$ with Lie nilpotence index $m$, then

$$\dim_F R \leq M(m+1,n)$$

where $M(m+1,n)$ is the maximum of $\frac{1}{2} \left( n^2 - \sum_{i=1}^{m+1} k_i^2 \right) + 1$ subject to the constraint $\sum_{i=1}^{m+1} k_i = n$ and $k_1, k_2, \ldots, k_{m+1}$ nonnegative integers. This answers in the affirmative a conjecture by the first and third authors. The case $m = 1$ reduces to a classical theorem of Schur (1905), later generalized by Jacobson (1944) to all fields, which asserts that if $F$ is an algebraically closed field of characteristic zero, and $R$ is any commutative $F$-subalgebra of $\mathbb{M}_n(F)$, then $\dim_F R \leq \left\lceil \frac{n^2}{4} \right\rceil + 1$. Examples constructed from block upper triangular matrices show that the upper bound of $M(m+1,n)$ cannot be lowered for any choice of $m$ and $n$. An explicit formula for $M(m+1,n)$ is also derived.

The Dudeney-Stockmeyer Conjecture, Andreas M. Hinz hinzm@math.lmu.de

Consider a variant of the famous Tower of Hanoi where moves of the discs are allowed along the edges of a connected graph $D$ whose $p := |D| \geq 3$ vertices represent the pegs of the puzzle. The task is to transfer a tower of $n \geq 1$ discs from one peg to another one according to the well-known rules of the classical game corresponding to $D = K_3$. Apart from its recreational appeal, the problem engenders fascinating integer sequences defined as the respective minimal number of moves.

For $p = 3$ there are two cases, $D = K_3$ and $D = K_{1,2}$, where these minimal sequences are given by the partial sums of powers of 2 and 2 times powers of 3, respectively, the latter, the sequence $2t_n := 3^n - 1$, is for the task to get from one peripheral vertex to the other. Minimality has long been open for the two corresponding cases for $p = 4$ though, namely $D = K_4$ (The Reve’s Puzzle).
and $D = K_{1,3}$ (the Star Tower of Hanoi). For both problems a solving strategy had been employed going back to Dudeney (1902) and Stewart (1941), leading to move numbers as partial sums of the sequences given by $k$-fold repetition of power $k$ of 2 and by 2 times the 3-smooth numbers, respectively, the latter leading to Stockmeyer’s sequence $s_n$. It was conjectured that these are indeed minimal as part of the Frame-Stewart Conjecture (1941) for the former and as Stockmeyer’s Conjecture (1994) for the latter. This was finally confirmed by Thierry Bousch in 2014 and 2017, respectively.

However, the center-to-periphery task for $D = K_{1,3}$ is still open. Applying the Dudeney-Stewart strategy leads to the sequence of move numbers $x_n$ given by the recurrence

$$
x_0 = 0, \forall n \geq 1 : x_n = \min\{x_m + s_m + t_{n-m} | 0 \leq m < n\}.
$$

But neither has this recurrence been solved explicitly, nor has minimality been proven. These findings are summarized in the following

**Conjecture:** Let $D$ be a connected graph with $|D| \geq 4$ and $i, j \in V(D)$, $i \neq j$.

We exclude the case where $D$ is a path and $i, j$ are its end vertices.

Then the task to transfer an $n$-tower, $n \geq 1$, from peg $i$ to peg $j$ in the minimum number of moves is solved by the Dudeney-Stewart strategy, namely

1. Transfer $m$ smallest discs, $0 \leq m < n$, from peg $i$ to some peg $k \in V(D \setminus P)$, where $P$ is an $i, j$-path in $D$ with $3 \leq |P| < |D|$.
2. Transfer $n - m$ largest discs from peg $i$ to peg $j$, avoiding peg $k$.
3. Transfer $m$ smallest discs from peg $k$ to peg $j$.
4. Minimize over the sum of the lengths of the three transfers and over $k$ and $m$.

The conjecture is supported by computations performed for small $n$ on all other connected graphs with $|D| = 4$ (paw, diamond, cycle, path), as well as the fork graph on 5 vertices. The exceptional case of a path $D$ and pendant vertices $i$ and $j$ is indeed special; for instance, for $|D| = 4$ and $n = 3$ the minimal solution has length 19, but no efficient algorithm is known which covers the general case.

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On uniformly continuous functions between pseudometric spaces and the Axiom of Countable Choice, Samuel G. da Silva samuel@ufba.br

In this work we show that the Axiom of Countable Choice is equivalent to two statements from the theory of pseudometric spaces: the first of them is a well-known characterization of uniform continuity for functions between (pseudo)metric spaces, and the second declares that sequentially compact pseudometric spaces are UC – meaning that all real valued, continuous functions defined on these spaces are necessarily uniformly continuous.

To be more precise: the following theorem includes all results of this paper. The two statements which are referred to in the previous paragraph are (ii) and (v).

**Theorem.** The following statements are equivalent:

(i) \( \text{AC}_\omega \) - the Axiom of Countable Choice, which declares that all countable families of non-empty sets have a choice function.

(ii) If \( M, N \) are pseudometric spaces and \( f : M \to N \) is a continuous function, \( f \) is uniformly continuous if, and only if, for every pair of (non-necessarily convergent) sequences \( (x_n), (y_n) \in M \) such that \( d_M(x_n, y_n) \to 0 \) one has

\[
d_N(f(x_n), f(y_n)) \to 0
\]

(iii) If \( M \) is a pseudometric space and \( f : M \to \mathbb{R} \) is a continuous function, \( f \) is uniformly continuous if, and only if, for every pair of (non-necessarily convergent) sequences \( (x_n), (y_n) \in M \) such that \( d_M(x_n, y_n) \to 0 \) one has \( |f(x_n) - f(y_n)| \to 0 \).

(iv) Let \( M, N \) be pseudometric spaces and assume \( M \) to be sequentially compact. Then, for every function \( f : M \to N, f \) is continuous if, and only if, it is uniformly continuous.

(v) Sequentially compact, pseudometric spaces are UC.

Hochschild cohomology of a Sullivan algebra, Jean Baptiste Gatsinzi Botswana International University of Science and Technology, Botswana jeangatsinzi@yahoo.fr

Let \( A = (\wedge V, d) \) be a minimal Sullivan algebra where \( V \) is finite dimensional.
We show that the Hochschild cohomology $HH^*(A; A)$ can be computed in terms of the differential graded Lie algebra of derivations of $A$. This provides another method to compute the loop space homology of a simply connected space for which $\pi_*(X) \otimes \mathbb{Q}$ is finite dimensional.

**Criterion for the basis property of the system of root functions of ordinary differential operators with regular boundary conditions,** Makhmud Sadybekov

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In this talk, we consider an operator $\mathcal{L}$ generated in $L_2(0, 1)$ by the ordinary differential expression

$$l(u) = u^{(2n)}(x) + q_1(x)u^{(2n-1)}(x) + \cdots + q_{2n}(x)u(x), \quad 0 < x < 1 \quad (1)$$

and the normalized boundary conditions

$$U_j(u) = \sum_{k=0}^{2n-1} \left[ \alpha_{jk}u^{(k)}(0) + \beta_{jk}u^{(k)}(1) \right] = 0, \quad j = 1, 2n. \quad (2)$$

We assume that the coefficients $q_k(x)$ in expression (1) are measurable essentially bounded functions, the forms $U_j(u)$ are linearly independent, and the boundary conditions (2) are regular according to Birkhoff, but not strong regular.

V.P. Mikhailov (1962) and G.M. Kesselman (1964) independently proved that under assumption of strong regularity of the boundary conditions (2), the system of root vectors of the operator $\mathcal{L}$ forms an unconditional basis in $L_2(0, 1)$. Only regularity of the boundary conditions for the basis property of the system of root vectors is not sufficient.

For the regular boundary conditions A.A. Shkalikov (1979) proved unconditional basis property with parentheses (two summands in each) in $L_2(0, 1)$.

V.A. Il’in (since 1994) founded the theory of the basis property of the system of root vectors in terms of the product of the norms of elements of biorthogonal systems.

In our talk we prove a new criterion for the basis property in $L_2(0, 1)$ of the system of root vectors of the operator $\mathcal{L}$, which does not use the construction of a biorthogonal system.
Existence Result for Impulsive Stochastic Partial Functional Differential Equations with Poisson jumps, Anguraj Annamalai angurajpsg@yahoo.com

We consider the impulsive stochastic partial functional differential equations with poisson jumps of the form

$$du(t) = Au(t)dt + g(t,u_t)dw(t) + \int h(t,u_t,x)\tilde{N}(dt,dx), \quad t \geq 0, t \neq t_k.$$  

$$\Delta u(t_k) = I_k(u_{t_k}), \quad k = 1, 2, \ldots.$$  

$$u_0(\theta) = \xi \in PC_\alpha, \theta \in [-\tau,0], a.s.$$  

We prove the existence and uniqueness of the equation by using fixed point theorem and the resolvent operator.

On branches of positive solutions to p-Laplacian problems at the extreme value of Nehari manifold method, Kaye Silva Universidade Federal de Goiás - UFG kayeoliveira@hotmail.com

This work concerns the application of the Nehari manifold method to the study of branches of positive solutions for the problem

$$-\Delta_p u = \lambda |u|^{p-2}u + f |u|^{\gamma-2}u, \quad u \in W_0^{1,p}(\Omega),$$

where $\Delta_p$ is the p-Laplacian operator, $f$ changes signs, $\lambda$ is a real parameter and $1 < p < \gamma < p^*$. A special care is given to the extreme value $\lambda^*$, which is characterized variationally by

$$\lambda^* = \inf \left\{ \frac{\int |\nabla u|^p}{\int |u|^p}, \quad u \in W_0^{1,p}(\Omega), \quad \int f |u|^{\gamma} \geq 0 \right\}.$$

The main result deals with the existence of two positive solutions when $\lambda \in (\lambda_1, \lambda^* + \varepsilon)$, where $\lambda_1$ is the first eigenvalue of $-\Delta_p$ and $\varepsilon > 0$. This is joint work with Yavdat Il’yasov (UFA-Russia).

Phase Transitions in the One-dimensional Coulomb Gas Ensembles with finite interaction range., Tatiana S. Turova tatyana@maths.lth.se
We consider the system of particles on a finite interval with pair-wise interaction and external force. We take into account interactions between nearest, second nearest, and so on, up till $K$-th nearest neighbours, where $K \geq 1$ is fixed arbitrarily.

This model was introduced by Malyshev to study the flow of charged particles on a rigorous mathematical level. It is a simplified version of a 3-dimensional classical Coulomb gas model. We study Gibbs distribution at finite positive temperature extending results on the zero temperature case (ground states). We derive the asymptotics for the mean and for the variances of the distances between the neighbouring charges. We prove that for any fixed $K$ depending on the strength of the external force there are several phase transitions in the local structure of the configuration of the particles in the limit when the number of particles goes to infinity. We identify 5 different phases for any positive temperature.

The proofs rely on a conditional central limit theorem for non-identical random variables, and on the correlation decay when $K > 1$, which has an interest on its own.

Stability of the Riemann solution for a strictly hyperbolic system of conservation laws with flux approximation, T Raja Sekhar trajasekhar@maths.iitkgp.ernet.in

In this work, we solve the Riemann problem for a strictly hyperbolic system of conservation laws under the linear approximation of flux functions with three parameters. The approximation does not change the Riemann solution to the original system. The Riemann problem cannot be solved for all combinations of piecewise constant initial states with classical elementary waves such as shock waves, rarefaction waves and contact discontinuities. For some initial data, the delta shock wave appears in the Riemann solution. A delta shock wave is a generalization of an ordinary shock wave, it consists of a discontinuity line plus a distributed Dirac delta function with the discontinuity line as its support. A delta shock wave is over-compressive in the sense that the number of characteristics entering the discontinuity line of the delta shock wave is more than in the case of an ordinary shock wave. We establish the generalized Rankine-Hugoniot jump condition for delta shock wave and the position, propagation speed and strength
of delta shock wave are computed. Lastly, we prove that the Riemann solution to the approximate system converge to the original system as the perturbation parameter tend to zero.

**Quasiconformal extension of meromorphic univalent functions with nonzero pole**, Goutam Satpati and Bappaditya Bhowmik  gsatpati@iitkgp.ac.in

It is well-known that the univalent functions defined in the unit disc that admit a quasiconformal extension to the extended complex plane play an important role in Teichmüller space theory. In this talk we consider meromorphic univalent functions \( f \) in the unit disc with a simple pole at \( z = p \in (0,1) \) which have a \( k \)-quasiconformal extension to the extended complex plane where \( 0 \leq k < 1 \). We denote the class of such functions by \( \Sigma_k(p) \). We first prove an area theorem for functions in this class. Next, we derive a sufficient condition for meromorphic functions in the unit disc with a simple pole at \( z = p \in (0,1) \) to belong to the class \( \Sigma_k(p) \). Finally, we also provide a representation formula for functions in \( \Sigma_k(p) \) and using this formula we derive an asymptotic estimate of the Laurent coefficients for the functions in this class.

**Canonical structures on generalized symmetric spaces and their applications**, Vitaly Balashchenko  balashchenko@bsu.by

Generalized symmetric spaces (in particular, homogeneous \( k \)-symmetric spaces) \( G/H \) admit a commutative algebra \( A(\theta) \) of canonical structures. The remarkable feature of these structures is that all of them are invariant with respect to both the Lie group \( G \) and the generalized ”symmetries” of \( G/H \).

Studied by many authors (V.I.Vedernikov, N.A.Steanov, A.Ledger, A.Gray, J.A.Wolf, A.S.Fedenko, O.Kowalski, V.Kac and others). Canonical structures of classical types such as almost complex \( (J^2 = -id) \), almost product \( (P^2 = id) \), \( f \)-structures of K.Yano \( (f^3 + f = 0) \) discovered in the 1990s by the author and N.A.Steanov on regular \( \Phi \)-spaces (in particular, on homogeneous \( k \)-symmetric spaces) play a remarkable role in this theory and its applications (e.g. they were
recently applied by I.Khemar to studying elliptic integrable systems). Specifically, these structures generate canonical invariant distributions on regular $\Phi$-spaces. We indicate algebraic criteria under which the base canonical distributions belong to the classes $\mathbf{F}$ (foliations), $\mathbf{AF}$ (anti-foliations), $\mathbf{TGF}$ (totally geodesic foliations) on homogeneous $k$-symmetric spaces endowed with the ”diagonal” Riemannian metrics. As a result, we obtain a wide collection of invariant Riemannian almost product structures for the Naveira classification, in particular, for the Reinhart foliations.

Further, we (jointly P.A.Dubovik) construct metric left-invariant $f$-structures on solvable and nilpotent Lie groups and study their relation to the generalized Hermitian geometry (e.g. nearly Kähler and Hermitian $f$-structures). In this respect, 2-step and some other nilpotent Lie groups represented as Riemannian homogeneous $k$-symmetric spaces are of special interest. Some general and particular examples including Heisenberg groups and their generalizations are considered.

In addition, we show that so-called ”metallic” structures (golden, silver and others) recently introduced by C.-E.Hretcanu and M.Crasmareanu can be effectively realized using the theory of canonical structures on homogeneous $k$-symmetric spaces.

Finally, we characterize base canonical $f$-structures on regular $\Phi$-spaces in the sense of nearly Kähler $f$-structures. As a corollary, it follows the recent result of the author and A.S.Samsonov that any base canonical $f$-structure on a naturally reductive homogeneous $k$-symmetric space is a nearly Kähler $f$-structure. In conclusion, we formulate some open problems in the directions above mentioned.

**On moments of Gamma exponentiated functional distribution**, Tibor K. Pogány *University of Rijeka, Croatia and Óbuda University, Hungary* poganj@pfri.hr

Reformulating the Gamma baseline distribution generating method by Zografos and Balakrishnan two kind of double integral expressions are derived for the raw moments of the Gamma exponentiated functional $\text{GE}(\alpha, h)$ distribution. Related probability distribution class is characterized in terms of Lambert $W$–
function.

Multi-twisted codes over finite fields and their dual codes, Varsha Chauhan and Anuradha Sharma varshac@iiitd.ac.in

Let $\mathbb{F}_q$ denote the finite field of order $q$, let $m_1, m_2, \cdots, m_\ell$ be positive integers satisfying $\gcd(m_i, q) = 1$ for $1 \leq i \leq \ell$, and let $n = m_1 + m_2 + \cdots + m_\ell$. Let $\Lambda = (\lambda_1, \lambda_2, \cdots, \lambda_\ell)$ be fixed, where $\lambda_1, \lambda_2, \cdots, \lambda_\ell$ are non-zero elements of $\mathbb{F}_q$. In this paper, we study the algebraic structure of $\Lambda$-multi-twisted codes of length $n$ over $\mathbb{F}_q$ and their dual codes with respect to the standard inner product on $\mathbb{F}_q^n$. We provide necessary and sufficient conditions for the existence of a self-dual $\Lambda$-multi-twisted code of length $n$ over $\mathbb{F}_q$, and obtain enumeration formulae for all self-dual and self-orthogonal $\Lambda$-multi-twisted codes of length $n$ over $\mathbb{F}_q$. We also derive some sufficient conditions under which a $\Lambda$-multi-twisted code is LCD. We determine the parity-check polynomial of all $\Lambda$-multi-twisted codes of length $n$ over $\mathbb{F}_q$ and obtain a BCH type bound on their minimum Hamming distances. We also determine generating sets of dual codes of some $\Lambda$-multi-twisted codes of length $n$ over $\mathbb{F}_q$ from the generating sets of the codes. Besides this, we provide a trace description for all $\Lambda$-multi-twisted codes of length $n$ over $\mathbb{F}_q$ by viewing these codes as direct sums of certain concatenated codes, which leads to a method to construct these codes. We also obtain a lower bound on their minimum Hamming distances using their multilevel concatenated structure.

Problems of stability ”in the large” of phase systems, Maksat Kalimoldayev and Assel Abdildayeva mnk@ipic.kz

In this paper we solve the problem of evaluation of attraction domains of stable equilibria states, phase based on new Lyapunov function, which expands the area of stability of the system, in contrast to the well-known Lyapunov function of the type ”kinetic energy plus potential energy”. Let’s consider the general model of multidimensional phase systems:

$$\frac{d\delta_i}{dt} = S_i, \quad \frac{dS_i}{dt} = w_i - D_i S_i - f_i(\delta_i) - \psi_i(\delta_i), \quad w_i = C_i^* x_i, \quad (3)$$
\[
\frac{dx_i}{dt} = A_i x_i + q_i S_i + b_i u_i, \quad i = 1, l, \tag{4}
\]

where the function \( \psi_i(\delta_*) = \sum_{k=1, k\neq i}^{l} P_{ik}(\delta_{ik}), \quad \delta_{ik} = \delta_i - \delta_k \) defines the relationship between subsystems and \( P_{ik}(.) \) – the given continuously differentiable periodic function \( D_i > 0 \) – damping factor. \( \delta_i \) – angular coordinate; \( S_i \) – angular velocity; \( x_i \) – state control vector; \( u_i \) – the feedback type control, \( f_i(\delta_i) = \frac{1}{\bar{I}_i} \left[ P_i \sin(\delta_{i0} + \delta_i) - P_i \sin \delta_i \right], \quad i = 1, l. \)

Investigation of stability "in the large" of the system (3), (4) will be carried out in the band \( G_{0i} = \{(\delta_i, S_i, x_i) \mid \delta_{-1i} < \delta_i < \delta_{0i}, \quad S_i \in R_1^1, \quad x_i \in R_n^1 \} \) using the method of Lyapunov. To investigate the stability "in the large", the feature \( v_{0i}(\delta_i, S_i) \) is offered which is defined in the band \( \overline{G_{0i}} \) as follows: \( v_{0i}(\delta_i, S_i) = \frac{1}{2} (S_i + \alpha_i D_i \delta_i)^2 + \frac{1}{2} \alpha_i D_i^2 (1 - \alpha_i) \delta_i^2 + F_i(\delta_i) + 2 D_i \sqrt{\alpha_i (1 - \alpha_i)} \tilde{F}_i(\delta_i) = \frac{1}{2} (S_i + \alpha_i D_i \delta_i)^2 + \int_0^{\delta_i} N_i(\delta_i) d \delta_i \)

where

\[
\alpha_i = \text{const} \quad (0 < \alpha_i < 1), \quad F_i(\delta_i) = \int_0^{\delta_i} f_i(\delta_i) d \delta_i,
\]

\[
\tilde{F}_i(\delta_i) = \int_0^{\delta_i} \sqrt{\delta_i f_i(\delta_i)} d \delta_i, \quad N_i(\delta_i)
\]

\[
= \alpha_i D_i^2 (1 - \alpha_i) \delta_i + f_i(\delta_i) + 2 D_i \sqrt{\alpha_i (1 - \alpha_i)} \sqrt{\delta_i f_i(\delta_i)}
\]

Functions \( F_i(\delta_i), \tilde{F}_i(\delta_i) \) are continuous in the strip \( \overline{G_{0i}} \).

On the basis of the developed theory, numerical calculations are carried out, which showed that the proposed method makes it possible to estimate quite accurately the regions of attraction of stable equilibrium states.

**Certain subclasses of harmonic univalent mappings**, Nirupam Ghosh and Allu Vasudevarao nirupamghoshmath@gmail.com

In this presentation, we introduce a new subclass \( \mathcal{B}^0_{H}(M) \), the class of sense preserving harmonic mappings \( f = h + \overline{g} \) in the unit disk \( \mathbb{D} := \{z \in \mathbb{C} : |z| < 1 \} \) satisfying \(|zh''(z)| \leq M - |zg''(z)|\). First we prove that for certain values of \( M, \mathcal{B}^0_{H}(M) \) is either included in the class of stable harmonic convex mappings or
in the class of stable harmonic starlike mappings. Next we solve the coefficient conjecture of Clunie and Sheil-Small for functions in $B^0_{\mathcal{H}}(M)$. We also prove the growth theorem, convolution and convex combination properties for functions in $B^0_{\mathcal{H}}(M)$. We provide the geometric structure of the image of unit disk $\mathbb{D}$ under the functions of $B^0_{\mathcal{H}}(M)$ and prove that each function $f$ in $B^0_{\mathcal{H}}(M)$ maps unit disk $\mathbb{D}$ onto a domain which is bounded by a rectifiable Jordan curve. Finally, we derive a sufficient condition for functions $f$ to belong to the class $B^0_{\mathcal{H}}(M)$ and as an application of it we construct harmonic univalent (i.e., one-to-one) polynomials belonging to $B^0_{\mathcal{H}}(M)$.

On the structure of the solution of integro-differential Bellman-Egorov equation in the synthesis problem of optimal control of oscillation processes, Akylbek Kerimbekov akl7@rambler.ru

The synthesis problem of optimal control is investigated for oscillation processes described by Fredholm integro-differential equations in this paper.

An optimal control problem is considered, where it is required to minimize the quadratic functional

$$I[u(t)] = \int_Q \left\{ [v(T,x) - \xi_1(x)]^2 + [v_r(T,x) - \xi_2(x)]^2 \right\} dx + \beta \int_0^T p^2[t,u(t)] dt, \beta > 0$$

on the set of solutions of boundary value problem

$$v_{tt} - Av = \lambda \int_0^T K(t,\tau)v(\tau,x)dx + g(t,x)f[t,u(t)], x \in Q, 0 < t \leq T,$$

$$v(0,x) = \psi_1(x), v_r(0,x) = \psi_2(x), x \in Q,$$

$$\Gamma v(t,x) \equiv \sum_{i,j} a_{ij} v_{x_j} \cos(v, x_i) + a(x)v = 0, x \in \gamma, 0 < t \leq T.$$
In determining the optimal control as a function of the state of the controlled process, i.e. \( u(t) = u[t, v(t, x)] \), according to the Bellman scheme and the method of Professor A.I. Egorov the following integro-differential equation is obtained:

\[
-\frac{\partial S[t, v(t, x), v_t(t, x)]}{\partial t} = \min_{u \in \mathbb{R}} \{ \beta p^2[t, u(t)] + \int_{Q} m_2(t, x) g(t, x) dx f[t, u(t)] + \int_{Q} \left( m_1(t, x) v_t(t, x) - m_2(t, x) \right) \int_{Q} D(t, x, y) v(t, y) dy \} dx + \lambda \int_{Q} m_2(t, x) \int_{0}^{T} K(t, \tau) v(\tau, x) d \tau dx
\]

with the additional condition

\[
S[T, v(T, x), v_t(T, x)] = \int_{Q} \left\{ [v(T, x) - \xi_1(x)]^2 + [v_t(T, x) - \xi_2(x)]^2 \right\} dx, \tag{6}
\]

where the vector function \( m(t, x) = \{ m_1(t, x), m_2(t, x) \} \)–is a gradient of the Bellman functional \( S[t, v(t, x), v_t(t, x)] \).

We investigate the problem of Cauchy-Bellman-Egorov (1) - (2), we define structure of solutions to control (1) and structure of synthesizing optimal control, and classes of functions \( f[t, u(t)] \) and \( p[t, u(t)] \) are indicated for which it is possible to lead the solution of the synthesis problem to numerical calculations.

**Brouwer’s conjecture on the Laplacian eigenvalues of a graph,** Sharieffuddin Pirzada pirzadasd@kashmiruniversity.ac.in

Let \( G(V, E) \) be a simple graph with \( n \) vertices and \( m \) edges having vertex set \( V(G) = \{ v_1, v_2, \ldots, v_n \} \) and edge set \( E(G) = \{ e_1, e_2, \ldots, e_m \} \). The adjacency matrix \( A = (a_{ij}) \) of \( G \) is a \((0, 1)\)-square matrix of order \( n \) whose \((i, j)\)-entry is equal to 1 if \( v_i \) is adjacent to \( v_j \) and equal to 0, otherwise. Let \( D(G) = \text{diag}(d_1, d_2, \ldots, d_n) \) be the diagonal matrix associated to \( G \), where \( d_i = \deg(v_i) \), for all \( i = 1, 2, \ldots, n \). The matrix \( L(G) = D(G) - A(G) \) is called the Laplacian matrix and its spectrum is called the Laplacian spectrum \((L\)-spectrum) of the graph \( G \). Let \( 0 = \mu_n \leq \mu_{n-1} \leq \cdots \leq \mu_1 \) be the \( L \)-spectrum.
of $G$ and let $S_k(G) = \sum_{i=1}^{k} \mu_i$, $k = 1, 2, \ldots, n$ be the sum of $k$ largest Laplacian eigenvalues of $G$ and let $d_i^*(G) = |\{v \in V(G) : d_v \geq i\}|$, for $i = 1, 2, \ldots, n$. Andries Brouwer conjectured that for a graph $G$ with $n$ vertices and $m$ edges, then for any $k$, $k = 1, 2, \ldots, n$,

$$S_k(G) = \sum_{i=1}^{k} \mu_i \leq m + \binom{k+1}{2}.$$ 

We discuss the progress on this conjecture, and show that Brouwer’s conjecture holds for some new classes of graphs.

Interaction of water waves with two submerged unequal plates with non-uniform permeability using Galerkin approximation, Sourav Gupta and Rupanwita Gayen  

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In this study, we analyze the effect of two unequal completely submerged permeable plates in the propagation of water waves under the assumptions of linear water wave theory. The permeability of the plates varies along the depth of submergence of the plates. The barriers are present in water of uniform depth. The velocity potential is expanded by using Havelock’s expansion of water wave potential. Employing Havelock’s inversion formula together with conditions on the permeable plates the boundary value problem is reduced to two coupled Fredholm type integral equations. A multi-term Galerkin approximation in terms of Chebyshev polynomials is used to solve the vector integral equations and to obtain numerical estimates for the reflection and the transmission coefficients. The effects of the length of the plates, the depth of submergence of the plates, the permeability of the plates and the separation length between them on the reflection coefficient are depicted graphically. The present results are validated against the known results for the case of two identical impermeable plates and for a single permeable plate.

On Leonard quadruples of Krawtchouk type, Jose Maria P. Balmaceda and Jryl
Let $V$ be a vector space of finite dimension over an algebraically closed field $K$ of characteristic zero. An ordered quadruple of linear transformations $A^{(1)}, A^{(2)}, A^{(3)}, A^{(4)}$ in $\text{End}(V)$ is a Leonard 4-tuple on $V$ if for each $k \in \{1, 2, 3, 4\}$ there is a basis for $V$ with respect to which the matrix representing $A^{(k)}$ is diagonal and the matrices representing the other three are irreducible tridiagonal. Leonard quadruples are generalizations of Leonard pairs which were discovered by Paul Terwilliger while working on the classification of $P$ and $Q$ polynomial association schemes. Leonard pairs provide a linear algebra interpretation of a theorem of Doug Leonard concerning Askey-type orthogonal polynomials. All Leonard pairs have been completely classified into 13 families. The classification of Leonard triples has also essentially been completed and is now known to fall into four types: (i) Krawtchouk; (ii) Racah; (iii) $q$-Racah; and (iv) Bannait/Ito. In this paper, we characterize the Leonard triples that can be extended to Leonard quadruples of Krawtchouk type and obtain a general form for Leonard quadruples of Krawtchouk type.

**Bounds on the transmission non-regularity index of unicyclic graphs**, Ezgi Kaya and Ayse D. Maden ezgi.kaya@igdir.edu.tr

Let $G$ be a graph. The distance between $u$ and $v$ in $G$ is defined as the length of a shortest path between them. The transmission of a vertex $u$, denoted by $T_G(u)$, is the sum of distances from it to all the other vertices in graph $G$. Therefore, the Wiener index of a graph $G$ is equal to half the sum of the transmissions of all vertices in $G$. $G$ is said to be transmission-regular if all its vertices have the same transmission, if not, $G$ is said to be transmission-non-regular. For a transmission-non-regular graph, how can we measure the extent of its transmission-non-regularity? The definition of the transmission-non-regularity index is $NT(G) = \sum |T_G(u) - T_G(v)|$, here the summation goes over all vertices of $G$. In this talk, we present lower and upper bounds on the transmission-non-regularity index of unicyclic graphs and characterize the extremal graphs.
Exponential stability of solutions to time-delay systems of neutral type, Inessa Matveeva matveeva@math.nsc.ru

We consider systems of delay differential equations of the form
\[
\dot{y}(t) = A(t)y(t) + B(t)y(t-\tau) + C(t)\dot{y}(t-\tau) + F(t, y(t), y(t-\tau), \dot{y}(t-\tau)), \quad t > 0, \tag{1}
\]
where \(A(t), B(t), C(t)\) are \((n \times n)\)-matrices with continuous \(T\)-periodic entries, \(\tau > 0\) is the time delay, \(F\) is a continuous vector function. We suppose that \(F(t, u, v, w)\) satisfies the Lipschitz condition with respect to \(u\) and the inequality
\[
\|F(t, u, v, w)\| \leq q_1\|u\|^{1+\omega_1} + q_2\|v\|^{1+\omega_2} + q_3\|w\|^{1+\omega_3}, \quad t \geq 0, \quad u, v, w \in \mathbb{C}^n,
\]
for some constants \(q_j, \omega_j \geq 0, j = 1, 2, 3\). This paper presents a continuation of our works on stability of solutions to time-delay differential equations. We establish conditions under which the zero solution to (1) is exponentially stable. These conditions are formulated in terms of Riccati type differential matrix inequalities. We obtain estimates characterizing exponential decay of solutions to (1) at infinity and estimates for attraction sets of the zero solution. These results are extended to time-delay systems with several delays. We use special Lyapunov–Krasovskii functionals and boundary value problems for the Lyapunov differential equation with periodic coefficients.

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Estimation of Exit Time in a Stochastic Predator-Prey Model, Murugan Suvinthra suvinthra@gmail.com

This work concerns the problem of extinction of prey population in the dynamical system of predator-prey interactions. The system is modeled as a stochastic differential equation with a nonlinear functional response, and the theory of large deviations and exit problems are applied to analyze the extinction of the prey species. The time of exit of the prey population from the desired domain is estimated by transforming the stochastic differential equation modeling predator-prey dynamics to an optimization or optimal control problem.

Exponential dichotomy for differential and difference equations with peri-
In the paper we consider systems of differential equations of the form

$$\frac{dy}{dt} = A(t)y + F(t,y), \quad -\infty < t < \infty,$$

where $A(t)$ is an $(n \times n)$-matrix with $T$-periodic entries. We suppose that exponential dichotomy holds for the linear systems. As well-known, this means that the spectrum of the monodromy matrix $Y(T)$ does not cross with the unit circle $\{\lambda \in \mathbb{C} : |\lambda| = 1\}$. We establish a criterion of exponential dichotomy in terms of a Hermitian matrix $H(t)$ and a projector $P$ such that

$$\begin{cases} \frac{d}{dt} H + HA(t) + A^*(t)H = -\left(Y^{-1}(t)\right)^* P^* Y^*(t) C(t) Y(t) PY^{-1}(t) \\ +(Y^{-1}(t))^* (I - P)^* Y^*(t) C(t) Y(t) (I - P) Y^{-1}(t), \quad 0 < t < T, \\ H(0) = H(T) > 0, \quad PY(T) = Y(T)P, \quad P^2 = P, \\ H(0) = P^* H(0) P + (I - P)^* H(0) (I - P). \end{cases}$$

This result is an analog of well-known theorems proved by M.G. Krein in the dichotomy problem for linear systems of differential equations with constant coefficients. In the case of exponential dichotomy we establish that (2) has a unique solution; moreover, $H(t) = H^*(t) > 0$ on $[0, T]$, the matrix $P$ is a projector onto the maximal invariant subspace of the monodromy matrix $Y(T)$ corresponding to the multipliers $\mu_j$ with $|\mu_j| < 1$. Using this result, we prove that (1) is uniquely solvable in the Sobolev space $W^1_2(\mathbb{R})$. This fact allows us to establish a theorem on exponential dichotomy for perturbed systems with periodic coefficients.

We prove analogous theorems for difference equations with periodic coefficients. This paper presents a continuation of our works on qualitative properties of solutions to differential and difference equations.

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Heating tokamak plasmas to very high temperatures can be achieved by many techniques, such as Joule effect, injection of neutrals and electromagnetic waves. The aim of this communication is to present a domain decomposition method to simulate the propagation of electromagnetic waves near the so-called lower hybrid (LH) frequency in a strongly magnetized cold plasma. Full wave simulations - i.e., the direct solution of Maxwell’s equations - of LH wave propagation are a challenging issue because of the short wave length with respect to the machine size, the non-homogeneity and non-Hermitianness of the plasma response tensor. We have developed a finite element algorithm using non-overlapping domain decomposition, which solves the time-harmonic Maxwell equations while explicitly enforcing the divergence condition. The antenna is modeled by essential or natural boundary conditions. The plasma response tensor involves a cold plasma approximation of the dielectric permittivity tensor. Expressions of the entries involve plasma frequencies, cyclotron frequencies of each species (ions and electron) and also the collision frequency. The plasma response tensor is complex and non-Hermitian as soon as collisions and Landau damping are considered. A mixed variational formulation is considered. This mixed formulation enforces the divergence condition on the electric field, and thus controls the space charge phenomena. In each subdomain the variational formulation is discretized using Taylor-Hood finite element. The interface conditions between sub-domains are dualised using Lagrange multipliers. This multi-domain formulation is well-posed and equivalent to the one-domain formulation. The full linear system involving all subdomains is a generalized saddle-point problem, where the matrix is a block sparse and non-Hermitian. The linear system is solved using a preconditioned GMRES algorithm. Numerical tests are described.

Mathematics e-Assessment Material Bank Abacus, Antti Rasila Aalto University, Helsinki, Finland antti.rasila@iki.fi

Abacus, based at Aalto University, is a project which aims to facilitate use of use computer aided assessment of mathematics and natural science assignments
in higher education. It started as collaboration of the seven Finnish technology universities but it has since evolved into a major international project. The consortium is open for educational institutions from any country. It is based on problems predominantly implemented by using the STACK automatic assessment system, which is a free and open source plugin for the popular Moodle learning management platform.

The Abacus project is motivated by the fact that a serious obstacle to the use of STACK has been the lack of available learning materials and associated support services. In this presentation, we give an overview of the STACK assessment system as well as discuss the history of the Abacus project, the present state of the material database and the collaboration network. We also outline our future plans for the project and opportunities for international collaboration.

Project website: http://abacus.aalto.fi

Measuring heterogeneity in an end-point-oriented cluster-based meta-analysis of individual patient data, Saskia Schiele and Andreas Brieden saskia.schiele@unibw.de

Meta-analysis is a commonly used statistical method in data analysis. In the field of medical research the combined evaluation of clinical drug studies is of special importance. Due to small amounts of participants in some clinical trials it is sometimes impossible to get statistically significant results. Meta-analysis helps increasing the underlying patient data set, leading to more reliable results. Even though meta-analysis is a widely used technique, it can lead to some problems. In many cases, for instance via Cochran’s $\chi^2$- Statistics, too much underlying heterogeneity is detected. This occurs even while using random-effects models, which encounter some inter-study heterogeneity.

One method to approach this problem is to perform an end-point oriented cluster analysis, developed by Brieden and Gritzmann, before doing the actual meta-analysis on all the given data points. Afterwards a separate meta-analysis is done on each of the resulting clusters. Due to the absence of global measures in this field, the resulting heterogeneity is so far only discussed based on the different values of Cochran’s $\chi^2$- statistics of each single cluster.

In this talk we propose several global indices primarily assessing the combined
heterogeneity of the conducted meta-analysis of each cluster, giving the possibility to compare different cluster-based meta-analyses regarding their heterogeneity. As the separation in clusters is only useful when dissimilar effects amongst these clusters can be observed, the proposed indices also include the heterogeneity between the clusters, leading to a single ratio evaluating the heterogeneity of the performed cluster-based meta-analysis. We discuss properties of the presented indices and compare them to each other. Finally, to apply the proposed indices, we present results of some recent data analysis on individual patient data, while showing that different data transformations, such as level-reduction of the input-data, can lead to better results regarding heterogeneity.

**China-Finland Joint On-line Course in Mathematical Modeling**, Xuxin Yang and Antti Rasila yangxx2002@sohu.com

We discuss a two years long (2016-2018) project for China-Finland collaboration in developing joint on-line courses for teaching mathematical modeling. The partners in the project have been Aalto University and Tampere University of Technology (coordinator) from Finland, and Hunan First Normal University and Shanxi University from China. The motivation for the project is that there is a long history of teaching of modeling on-line as a collaborative effort in Finnish universities. The Chinese universities participating in the project also have a strong background in teaching mathematical modeling.

The main goal of the project has been to develop a joint e-learning course in mathematical modeling, initially focusing on models involving ordinary and partial differential equations. The project also includes other related activities such as student and faculty exchanges. Pilot course was successfully implemented in Autumn 2017 with both Chinese and Finnish university students working together online. In this presentation, we outline results and general experiences from the project.

**Domino tilings of rectangle with one deleted square**, Askar Dzhumadil’daev and Radmir Sultamuratov dzhuma@hotmail.com
Domino tilings problem is also known as a dimer problem or matchings in graph $P_m \times P_n$. Research on dimers in statistical mechanics had a major breakthrough in 1961, when Kasteleyn (and, independently, Temperley & Fisher) discovered a Pfaffian method to count the matchings of subgraph of the infinite square lattice and introduced remarkable formula

$$
\prod_{j=1}^{\lfloor \frac{m}{2} \rfloor} \prod_{k=1}^{\lfloor \frac{n}{2} \rfloor} \left(4\cos^2 \frac{\pi j}{m+1} + 4\cos^2 \frac{\pi k}{n+1}\right),
$$

which calculates the number of all possible domino tilings of $m \times n$ rectangle, for case when $mn$ is even.

In the case $mn$ is odd, there is no perfect matching, and we set ourselves the goal to study the number of all possible maximum matchings. Maximum matching is a matching that contains the largest possible number of edges. In other words, we consider domino tilings of rectangle with one deleted square.

D. Klarner and J. Polack gave method on calculating domino tilings of rectangles with fixed width and this method differs completely with the known Pfaffian method. We have improved this method for the case with deleted square. Moreover, it is applicable even if several squares are deleted. Also, we show recurrence relations of incidence matrices, which were introduced by D. Klarner and J. Polack, and bring and use additional incidence matrices.

Ultraparabolic equations with a noninvertible operator coefficients in front of time variables, Aleksandr I. Kozhanov kozhanov@math.nsc.ru

In the presentation we expose our results on solvability of boundary value problems for the differential equations

$$L_\alpha u_t + L_\beta u_a - Mu = f(x,t,a),$$

where $t \in (0,T)$, $a \in (0,A)$, $x \in \Omega \subset \mathbb{R}^n$, $L_\alpha$ and $L_\beta$ are the differential operators

$$L_\alpha = \alpha_0(t,a) + \alpha_1(t,a)\Delta, \quad L_\beta = \beta_0(t,a) + \beta_1(t,a)\Delta,$$
with $\Delta$ the Laplace operator and $M$ a second order elliptic operator both in the space variables. The peculiarities of the problem are the facts that we do not require the invertibility of $L_\alpha$ and $L_\beta$, the statement of the problem depends on the functions $\alpha_0(t,a), \alpha_1(t,a), \beta_0(t,a)$, and $\beta_1(t,a)$, and the semigroup or degenerate semigroup theory is not applicable here. For the problems proposed we establish existence and uniqueness theorems of regular solutions (i.e., having all Sobolev generalized derivatives occurring into the equation). Some generalizations and refinements of the results are discussed.

**Quasi-compactness of transfer operators for weakly regular expanding maps**, Daniel Smania and Alexander Arbieto

Quasi-compactness of transfer operators is one of the main tools to study the ergodic theory of smooth dynamical systems. To this end it is necessary to find Banach spaces of functions on which the action of the transfer operator is well-behaved (it satisfies the Lasota-Yorke inequality. See the works of Lasota and Yorke and Ionesco Tulcea and Marinescu).

For smooth expanding maps of the circle one can just take a space of Hölder continuous functions. However when the regularity of the maps is weaker, as piecewise expanding maps on the interval and Lorenz maps, to find a suitable function space is a tricky business. There is a long list of developments in the pursuit of them by Hofbauer and Keller, Rychlik Keller, Saussol, Liverani, Butterley, Thomine and others.

We propose a new class of Banach spaces of functions, with a surprisingly simple definition, where the action of transfer operators for expanding maps is quasi-compact, even in settings where either the phase space is irregular (as for instance in Julia sets of hyperbolic polynomials and symbolic spaces), or the map is irregular ($C^{1+\alpha}$-piecewise expanding maps and Lorenz maps).

Moreover these function spaces contains unbounded observables and indeed very general observables. Finally in certain situations these spaces coincides with familiar spaces of functions, for which there is a large literature available.
Polytope of integer partitions of $n$ is the convex hull of the corresponding integer points in the $n$-dimensional space. Of special importance are its vertices and their number since every partition is a convex combination of some vertices. The difficulty in their study is that no criterion for vertices is known, though recognizing vertices is proved to be a polynomially solvable problem.

Computations show that the number of vertices does not grow up monotonously: it drops down at every even $n$ and its peaks at prime $n$’s are higher than others. We show that the saw-toothed shape of this function may be caused by the large number of partitions of even $n$, which were studied and counted by N. Metropolis, and that most of partitions are of this kind. Metropolis partitions belong to the class of partitions that are convex combinations of two other partitions. We reveal that divisibility of $n$ by 3 also reduces the number of vertices and explain this by a large number of partitions that are convex combinations of three but not two others. We characterize the coefficients in such representations for arbitrary integral polytope.

To approach the prime $n$ phenomenon, we classify integer numbers by their proximity, in a certain sense, to primes and visually demonstrate stratification of the vertex number function to layers corresponding to these classes. Our main conjecture claims that the number of vertices depends on the sets of small divisors of $n$.

Discovered dependence of the number of vertices of the integer partition polytope on multiplicative properties of the partitioned number gives rise to variety of interesting problems. Some of them are concerned with the almost untouched support vertices, those from which all other partitions can be built using two simple combinatorial operations.

**An expansion for the number of partitions of an integer**, Stella Brassesco and Arnaud Meyroneinc sbrasses@gmail.com

We consider $p(n)$ the number of unrestricted partitions of a natural number $n$, starting from a formula derived by Luis Báez-Duarte [Advances in Mathemat-
ics 125,1 (1997)] by relating its generating function $f(t)$ with the characteristic functions of a family of infinitely divisible random variables indexed by $t$. The asymptotics of $p(n)$ follows then from a local central limit theorem as $t \uparrow 1$ suitably with $n$. We take further that analysis and compute asymptotic expansions as $t \uparrow 1$ of the terms in that formula, to obtain the following expansion for $p(n)$: for each given $N > 0$,

$$p(n) = \frac{2\pi^2}{3\sqrt{3}} e^{\sqrt{\frac{2\pi^2}{3}(n-\frac{1}{24})+\frac{1}{4}}} \left\{ 1 - \sum_{\ell=1}^{N} \frac{D_\ell}{(1 + 2c_n^2)^\ell} \right\} + \mathcal{R}_{N+1}.$$

The terms $c_n^2$ depend only on $n$, are defined in terms of the first two cumulants of the random variables, and satisfy $c_n^2 \asymp \sqrt{\frac{2\pi^2}{3}(n-\frac{1}{24})+\frac{1}{4}}$ (asymptotically), while the coefficients $D_\ell$ are positive and have simple expressions as finite sums of binomial coefficients, and the remainder $\mathcal{R}_{N+1} = o(n^{N/2})$.

The proofs are based on the derivation of simple asymptotic formulae for the logarithm of the generating function of the partitions and for all the cumulants that are of independent interest. They approximate them very accurately as $t \uparrow 1$, in the sense that the difference vanishes faster than any positive power of $(1 - t)$. Along the way, we obtain explicit expressions for the latter as series in terms of the Eulerian polynomials.

**On quality of convergence of power normalized partial maxima of iid random variables**, Sreenivasan Ravi, Deepesh Bhati and A. S. Praveena *University of Mysore, India* ravi@statistics.uni-mysore.ac.in

Extreme value laws are non-degenerate limit laws of partial maxima of independent identically distributed (iid) random variables under linear normalization. These are well known. If the normalization is power normalization, a non-linear normalization, then the non-degenerate limit laws are the $p$-max stable laws. In this article, we look at

(i) Hellinger distance and variational distance between the distribution function (df) of the power normalized partial maxima and its limit df,
(ii) rate of convergence under the uniform metric and total variation metric, extending the idea of second-order regular variation to slowly varying functions, and

(iii) some diagnostic plots to ascertain the max domains under power normalization.

Commutativity Conditions in Banach Algebras, Cheikh O. Hamoud Department of Mathematics and Science, Ajman University, Ajman Campus, United Arab Emirates c.hamoud@ajman.ac.ae

We show that unital complex Banach algebras are commutative under certain conditions. The extension of the results to nonunital algebras and to wider classes of topological algebras is also considered.

C. Le Page inaugurated the study of metric and algebraic conditions implying commutativity in Banach algebras in 1967. Since then, many characterizations for different levels of commutativity in Banach (and topological) algebras were obtained by several authors. In addition to their possible direct applications, these characterizations uncover interesting properties of commutative Banach algebras. This note prolongs the study of commutativity in Banach algebras. Using simple algebraic methods and also classical methods of complex analysis, we obtain several characterizations of commutativity in complex Banach algebras and in complete locally multiplicatively convex algebras.

An Improved Mathematical Formulation for the Chemical Compositional Simulation, Zharasbek Baishemirov, Abdumauvlen Berdyshev and Bakhbergen Bekbauov zbai.kz@gmail.com

The energy balance equation is derived by assuming that energy is a function of temperature only and energy flux in the aquifer or reservoir occurs by advection and heat conduction only. We write the energy conser-
vation equation in terms of the modified porosity, as opposed to the existing chemical compositional model formulation.

This new mathematical formulation of species conservation equations makes it possible to apply a sequential solution approach to solve each of these equations separately and implicitly for the total concentration of each component in fluid phases. A flash calculation is then performed to obtain the phase saturations and the concentrations of components in each phase. The linear, Freundlich or Langmuir adsorption isotherm model can be applied to calculate the concentration of the adsorbed component. Numerical values can be most simply calculated as the weighted averages obtained from previous time-step values.

The sequential solution procedure is carried out in the following order: (a) solution of the pressure equation implicitly, (b) solution of the transport system implicitly for the total concentration of each component, and (c) solution of the energy conservation equation implicitly.

100 years of Hardy inequalities: critical cases, extensions, remainders, stability and superweights, Durvudkhan Suragan suragan@math.kz

In this talk we give an extension of the classical Caffarelli-Kohn-Nirenberg inequalities. Moreover, we also obtain anisotropic versions of these inequalities which can be conveniently formulated in the language of Folland and Stein’s homogeneous groups. Consequently, we obtain remainder estimates for $L^p$-weighted Hardy inequalities on homogeneous (Lie) groups, which are also new in the Euclidean setting of $\mathbb{R}^n$. The critical Hardy inequalities of logarithmic type and uncertainty type principles on homogeneous groups are obtained. Moreover, we investigate another improved version of $L^p$-weighted Hardy inequalities involving a distance and stability estimates. The relation between the critical and the subcritical Hardy inequalities on homogeneous groups is also investigated. We also establish sharp Hardy type inequalities in $L^p$, $1 < p < \infty$, with superweights. There are two reasons why we call the appearing weights the superweights: the arbitrariness of the choice of any homogeneous quasi-norm and a wide range of parameters. This talk is based on our recent papers with Michael Ruzhansky.
The Alekseevskii conjecture in low dimensions, Romina M. Arroyo and Ramiro A. Lafuente romina.arroyo@gmail.com

One of the most important open problems on Einstein homogeneous manifolds is the Alekseevskii conjecture. This conjecture says that any connected homogeneous Einstein space of negative scalar curvature is diffeomorphic to a Euclidean space.

The aim of this talk is to show that the Alekseevskii conjecture holds up to dimension 8 (excluding 5 possible exceptions), and also in dimensions 9 and 10 provided that the transitive group is not semisimple.

This talk is based on a joint work with Ramiro Lafuente.

Oscillation Criteria For The Higher Order Nonlinear Dynamic Equation on Time Scales with Variable Coefficients, Shekhar S. Negi, Syed Abbas and Muslim Malik shekharsinghnegi2017@gmail.com

In this paper, we establish some sufficient conditions for the existence of oscillatory solutions of higher order nonlinear dynamic equation on time scales with the variable coefficients. At the end, for an application, we present some examples which illustrate our analytic findings.

Optimal Controls of Stochastic Integrodifferential Equations with Non-instantaneous Impulses, Rajesh Dhayal, Muslim Malik and Syed Abbas dhayalrajesh2010@gmail.com

The purpose of this paper is to study the existence of mild solutions for a class of stochastic integrodifferential equations with non-instantaneous impulses in Hilbert space. Also, we briefly discuss the existence of optimal control for stochastic integrodifferential equations. The results are obtained by using the stochastic analysis theory, semigroup theory, and Krasnoselskii’s fixed point theorem. Finally, we have given an example to illustrate the application of these proposed results.
An Erdös-Ko-Rado type theorem for 1-separated k-sets in several circles, Adrian Pastine and Emiliano J. Estrugo adrian.pastine.tag@gmail.com

A set of $k$ ordered objects that does not contain elements at certain distances $S = \{d_1, d_2, \ldots, d_s\}$ is called $S$-separated $k$-set. Kaplansky (1943), Konvalina (1981), and Mansour and Sun (2007), studied how many separated $k$-sets are ordered in different ways. Kaplansky studied $\{1\}$-separated $k$-sets ordered in a circle or line. Konvaline studied the amount of $\{1\}$-separated $k$-sets in two circles or lines of the same size. Mansour and Sun studied how many $\{m-1, 2m-1, \ldots, pm-1\}$-separated sets are in a circle or line. In (2003) Talbot proved an Erdös-Ko-Rado type theorem for intersecting families of $\{1, \ldots, s\}$-separated $k$-sets in a circle. This is a Theorem that studies the size of maximum intersecting families, and characterize them. In this work, we study an Erdös-Ko-Rado type theorem for $\{1\}$-separated intersecting families and the amount of $\{1\}$-separated $k$-sets in several circles of different sizes.

The concentration compactness principle for $p$-Fractional Laplacian in unbounded domains, Analía C. Silva and Julian F. Bonder analia.silva82@gmail.com

The famous concentration-compactness principle (CCP) due to Lions in 80’s is the key to solve problems with lack of compactness in Sobolev embeddings. This principle was originally formulated for critical problems in bounded domain and later extended to deal with critical problem in unbounded domains by Chabrowski, is of prime importance since it describes the concentration by a weighted sum of Dirac masses and the loss of mass by measures ”supported at infinity”. In this talk we show an extension of the refined concentration-compactness at infinity for a nonlocal operator (Fractional Laplacian) and its applications to prove existence of solutions for critical equations in unbounded domains. Joint work Julián Fernández Bonder (UBA-IMAS) and Nicolas Saintier (UBA-IMAS).

A survey on sums of element orders in finite groups, Mohsen Amiri m.amiri77@gmail.com
Given a finite group $G$, write $\psi(G)$ to denote the sum of the orders of the elements of $G$. In this article, I give a survey about this function.

**Adjoint Riemann-Liouville and Caputo fractional differential operators**, Zeynep Kayar *Van Yüzüncü Yıl University, Turkey* zykayar@gmail.com and zeynepkayar@yyu.edu.tr

The adjoints of Riemann-Liouville and Caputo fractional differential operators are obtained by using Lagrange Identity. Then associated to these operators, $\alpha$-order ($1 < \alpha < 2$) fractional differential equations are established. After defining boundary conditions of such equations, the adjoint boundary conditions are presented. Then the related eigenvalue problem is investigated.

**Symmetry results in the half space for a semi-linear fractional Laplace equation through a one-dimensional analysis**, Leandro M. Del Pezzo ldelpezzo@utdt.edu

In this talk we will analyze the semi-linear fractional Laplace equation

$$(-\Delta)^s u = f(u) \quad \text{in } \mathbb{R}^N_+, \quad u = 0 \quad \text{in } \mathbb{R}^N \setminus \mathbb{R}^N_+,$$

where $\mathbb{R}^N_+ = \{x = (x', x_N) \in \mathbb{R}^N : x_N > 0\}$ stands for the half-space and $f$ is a locally Lipschitz nonlinearity. We will completely characterize one-dimensional bounded solutions of this problem, and we will prove among other things that if $u$ is a bounded solution with $\rho := \sup_{\mathbb{R}^N} u$ verifying $f(\rho) = 0$, then $u$ is necessarily one-dimensional.

**Distributions, permutations, and quasigroups: Specifying exact structures**, Bokhee Im and Jonathan D. Smith *Department of Mathematics, Chonnam National University, Gwangju 61186, Republic of Korea. and Department of Mathematics, Iowa State University, Ames, Iowa 50011, U.S.A.* bim@jnu.ac.kr

Suppose that $n$ is an integer greater than 1. The simplex $\Delta_{n-1}$, Birkhoff polytope $\Omega_n$ and Latin square polytope $\Lambda_n$ specify a projective geometry obtained by
identifying antipodal points on a sphere bounding a ball centered at the barycenter of the polytope. We investigate sufficient conditions for homogeneous coordinates of points in these projective geometries to locate exact vertices of the respective polytopes.

**Shape Optimization Approach to the Bernoulli Problem: A Lagrangian Formulation**, Julius Fergy T. Rabago and Jerico B. Bacani jfrabago@gmail.com

The exterior Bernoulli free boundary problem is reformulated into a shape optimization setting by tracking the Dirichlet data. The shape derivative of the corresponding cost functional is established through a Lagrangian formulation coupled with the velocity method. A numerical example using the traction method or $H^1$ gradient method is also provided.

**The infinite tame-wild dichotomy and Brauer-Thrall 3 conjectures**, Miodrag C. Iovanov Miodrag Cristian Iovanov and The University of Iowa miodrag-iovanov@uiowa.edu

The famous tame-wild dichotomy is a fundamental result in the representation theory of finite dimensional algebras, due to Drozd. For infinite dimensional algebras and categories of finite dimensional rational modules (comodules), it is an open question (and conjecture), due to Daniel Simson, whether the tame-wild dichotomy still holds. This has been approached by various authors over the past few years and proved in certain particular cases. We settle this conjecture in the positive in its full generality. The key part of the approach is proving new representation theoretic characterizations for when the Ext quiver of the category of finite dimensional representations of an arbitrary algebra $A$ (or more generally, of the comodules over a coalgebra) is locally finite (i.e. $\dim(\text{Ext}^1(S, T)) < \infty$ for all simple finite dimensional $A$-modules $S, T$); this is so exactly when for every dimension vector $d$, the representations of $A$ of dimension vector $d$ are all contained in a finite subcategory (a category of modules over a finite dimensional quotient algebra). This allows one to reduce checking the tame/not wild properties for finite dimensional algebra quotients of $A$, where they are equivalent by
Drozd’s classical result. We also show that these two properties are local in the sense of (Gabriel-Grothendieck) localization: a category of comodules is tame/not wild if and only if every “finite” localization is so. We use the methods and various embeddings we obtain, to give a proof for another conjecture in representation theory sometimes called “the Brauer-Thrall 3 conjecture”, also raised by Simson, for the class of all wild algebras (which covers “almost all” algebras): we prove that over such algebras, there are indecomposable representations of arbitrarily large (infinite) dimension.

On knots with half-integral toroidal surgeries, Mario Eudave-Muñoz Instituto de Matemáticas, Universidad Nacional Autónoma de México, Ciudad Universitaria, Ciudad de México 04510, México mario@matem.unam.mx

Let $K$ be an hyperbolic knot in the 3-sphere. By results of W. Thurston, all Dehn surgeries on $K$, except a finite number, produce hyperbolic manifolds. Those surgeries producing non-hyperbolic manifolds are called exceptional. Among the exceptional surgeries, are those producing toroidal manifolds, that is, manifolds which contain an incompressible torus. Gordon and Luecke proved that if $p/q$-Dehn surgery on $K$ produce a toroidal manifold, then $|q| \leq 2$. Later, I constructed an infinite family of knots, denoted $k(\ell, m, n, p)$, such that for some non-integral slope $r/2$, we have that $r/2$-Dehn surgery produces a toroidal manifold. Some years later, Gordon and Luecke proved that if $r/2$-Dehn surgery on a hyperbolic knot produces a toroidal manifold, then in fact $K$ is one of the knots $k(\ell, m, n, p)$. So, the determination of all knots with a half-integral toroidal surgery is complete. The case of toroidal integral surgeries is more complicated; there are so many examples, but there is no a classification. The knots $k(\ell, m, n, p)$ were constructed by means of the Montesinos trick, that is, by using tangles and double branched covers. In this report we now give an explicit construction of these knots. Each of these knots contains a twice punctured torus properly embedded in its exterior, whose boundary consists of curves with half-integral slope. In the new construction we can see the knots and the corresponding twice punctured torus.
Interactions of cryptography and representation theory - the matrix of rotation symmetric Boolean functions, Lavinia C. Ciungu and Miodrag C. Iovanov
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In the study of rotation symmetric Boolean functions (RSBF) in cryptography, it is natural to consider the equivalence of Boolean vectors in $F_2^n$ given by $v \sim w$ if $w$ is obtained from $v$ by cyclic permutation (rotation). Several authors (Clark, Cusick, Hell, Maitra, Maximov, Stănică), in relation to RSBF’s, considered the square matrix $n \cdot A$ obtained as follows: let $(G_i)_{i=1, \ldots, g_n}$ be the equivalence classes of this relation $\sim$ and $\Lambda_i$ be representatives; the entries of $n \cdot A$ are $(\sum_{x \in G_i} (-1)^{x \cdot \Lambda_j})_{i,j}$. Some properties of this matrix were obtained for $n$ odd in the literature. We obtain a few new formulas regarding the number of classes of various types, and investigate the matrix $n \cdot A$ in general. One of our main results is that $n \cdot A$ satisfies $(n \cdot A)^2 = 2^n \cdot \text{Id}$, and it is conjugate to its transpose by a diagonal matrix. This is not an immediate consequence of the similar property of the related Hadamard type matrix $(p_{v,w})_{v,w \in F_2^n} = ((-1)^{v \cdot w})_{v,w}$, but it is rather connected to character theory. We show that the entries of the matrix $n \cdot A$ are essentially the character values of the irreducible representations of the semidirect (or wreath) product of $F_2^n \rtimes C_n$, where $C_n$ is the cyclic group with $n$ elements, which yields further properties of this matrix. This connection suggests possible future investigations, and motivates the introduction, in cryptographic applications, of Boolean functions with various other types of symmetry.

Meridian Surfaces with Parallel Normalized and Constant Mean Curvature in Pseudo-Euclidean 4-Space with Neutral Metric, Betul Bulca and Velichka Milousheva
Uludag University, Turkey, Bulgarian Academy of Sciences and L. Karavelov Civil Engineering Higher School, BULGARIA bbulca@uludag.edu,trandvmil@math.bas.bg

We consider a special class of Lorentz surfaces in the 4-dimensional pseudo-Euclidean space with neutral metric which are one-parameter systems of meridians of rotational hyper surfaces with timelike, spacelike or lightlike axis and call them
meridian surfaces. We give the complete classification of minimal and quasiminimal meridian surfaces.

We also classify the meridian surfaces with non-zero constant mean curvature and parallel normalized mean curvature vector.

**Inverse Problem for the Second Zagreb Index, Forgotten Index, Hyper-Zagreb Index and Sigma Index**, Aysun Yurttas, Muge Togan and Ismail Naci Cangul *Uludag University, Turkey* ayurttas@uludag.edu.tr, capkinm@uludag.edu.tr, andncangul@gmail.com

The inverse problem for topological graph indices is about the existence of a graph having its index value equal to a given non-negative integer. In this paper, we consider the problem for second Zagreb index $M_2(G)$, forgotten index $F(G)$ and hyper-Zagreb index $HM(G)$. We will first show that $M_2(G)$ can take all positive integer values except 2, 3, 5, 6, 7, 10, 11, 13, 15 and 17, and $F(G)$ can take all positive even integer values except for 4, 6, 8, 12, 14, 16, 20, 22, 28 and 36. We will determine which values $HM(G)$ can take. We also show that $\sigma(G)$ must be even for any graph $G$, and we construct some graph classes such that the sigma indices of them covers all positive even integers.

**An Application of Wave Packet Transform to Scattering Theory**, Taisuke Yoneyama *Tokyo University of Science* t.yoneyama226@gmail.com

In this talk, we shall consider Schrödinger equation with time-dependent short-range potentials by introducing wave packet transform. Wave packet transform is defined by A. Córdoba and C. Fefferman. We will give the outline of the proof of the existence of the wave operators by wave packet transform and explain the difference between the characterization space in our study and that in H. Kitada–K. Yajima. In the proof, we use the representation with wave packet transform, which is developed by K. Kato, M. Kobayashi and S. Ito.

**Well-Rounded Lattices from $\mathbb{Z}$-modules of Cyclotomic Fields via the Minkowski Embedding**, Robson R. Araujo and Sueli I.R. Costa
Lattices are discrete additive subgroups of $\mathbb{R}^n$. Using the Minkowski embedding (or canonical homomorphism) it is possible to obtain lattices in $\mathbb{R}^n$ from $\mathbb{Z}$-modules of full rank in a number field $\mathbb{K}$ of degree $n$, which are called algebraic lattices. The algebraic properties of these lattices allows to determine important data such the minimum distance, the packing density and the product distance which are used in lattice coding for transmission over Gaussian and Rayleigh channels.

A lattice $\Lambda \subset \mathbb{R}^n$ is called well-rounded (WR-lattice) if the set of minimal vectors of $\Lambda$ contains $n$ linearly independent vectors. Recently, well-rounded lattices have been recommended for transmissions in Rayleigh Fading SISO Channels and MIMO Wiretap Channels.

A paper of 2012 published by L. Fukshanksky and K. Petersen shows an interesting relationship between algebraic lattices (in that paper called ideal lattices) and well-rounded lattices. If $\sigma$ denotes the canonical embedding in a number field $\mathbb{K}$ and $\mathcal{O}_\mathbb{K}$ denotes its ring of integers, that paper proves that the lattice $\sigma(\mathcal{O}_\mathbb{K})$ is well-rounded if and only if $\mathbb{K}$ is a cyclotomic field.

Complementary, in this work we prove that there exists infinitely many well-rounded algebraic lattices in $\mathbb{R}^p$, where $p$ is an odd prime number. For that, we choose a prime number $q$ such that $q \equiv 1 \ (mod \ p)$ and a number field $\mathbb{K}$ of degree $p$ contained in the cyclotomic field $\mathbb{Q}(\zeta_q)$ whose conductor is $q$. Denote by $\theta$ the generator of the Galois group of $\mathbb{K}$ and consider $t$ the trace of $\zeta_q$ in the extension $\mathbb{Q}(\zeta_q)/\mathbb{K}$. Inside $\mathbb{K}$ we use a $\mathbb{Z}$-module

$$M_m = \left\{ \sum_{i=0}^{p-1} a_i \theta^i (t) \in \mathcal{O}_\mathbb{K} : \sum_{i=0}^{p-1} a_i \equiv 0 \ (mod \ m) \right\}$$

for some positive integer $m$ number satisfying $m \equiv 1 \ (mod \ p)$ to obtain the well-rounded lattice $\sigma(M_m)$ in $\mathbb{R}^p$. Some complements and explicit examples are also derived.

**Singularities and global solutions in the Schrodinger Hartree equation**, Anudeep K. Arora *The George Washington University, Washington, DC* anudeep@gwu.edu

We consider a nonlinear Schrödinger type equation with nonlocal nonlinear-
ity, of a convolution type, called the generalized Hartree equation. In the focusing case we investigate global behavior of solutions and formation of stable singularities. In the inter-critical regime we first obtain a dichotomy for global vs finite time existing solutions exhibiting two methods of obtaining scattering: one via Kenig-Merle concentration - compactness and another one is using Dodson-Murphy approach via Morawetz on Tao’s scattering criteria. Next, we investigate stable blow-up solutions in a critical regime and describe the blow-up dynamics, which is similar to NLS.

An inverse problem in KdV equation with over determination data, Gnanavel Soundararajan and Kumarasamy Sakthivel gnanavel.math.bu@gmail.com

In this work, we study an inverse problem of reconstructing a space dependent coefficient in a generalized Korteweg–de Vries (KdV) equation arising in physical systems with variable topography from the overdetermination data. The identification problem is transformed into an optimization problem and existence of a minimizer for the cost functional is established. Then we prove a stability estimate for retrieving the unknown coefficient in KdV equation with the upper bound of given measurements.

Application of methods of complex functions in nonlinear problems of elasticity of composite materials with interface cracks, Tatiana O. Domanskaia and Veniamin M. Malkov tanyath57@gmail.com

The exact analytical solutions of the nonlinear problems (plane-strain and plane-stress) of elasticity for bi-material plane with an interface crack have been obtained. The plane is formed by joining of two half-planes made from different materials. Mechanical properties of half-planes are described with the model of John’s harmonic material. The application of this model has allowed using the methods of the complex functions in the nonlinear boundary value problems. The stresses and displacements are expressed through two analytical functions of complex variable, which are defined from nonlinear boundary conditions. As particular case the problem is solved for the plane with a free interface crack at given
constant nominal (Piola) stresses at infinity. The expressions for nominal stresses, Cauchy stresses and displacements are obtained. From the general solutions the asymptotic expansions of these functions have been constructed in vicinities of crack tips. In nonlinear problem of uniaxial extension of a plane with free crack it is established, that the formulas which give the crack disclosing and the stress intensity factors (SIF) near the crack tips coincide completely with the similar formulas derived from the equations of the linear elasticity. The nominal stresses have the root singularity at the crack tips; the Cauchy stresses have no singularity. The solution of problem of interface crack has an oscillation at the vicinities of crack tips.

**On the Generalized Wintgen inequality,** İrem K. Erken, Ruken Görünüş and Cengizhan Murathan  irem.erken@btu.edu.tr, rukengorunus16@gmail.com and cengiz@uludag.edu.tr

The generalized Wintgen inequality was conjectured by De Smet, Dillen, Verstraelen and Vrancken in 1999 for submanifolds in real space forms. It is also known as the DDVV conjecture. It was proven recently by Lu (2011) and by Ge and Tang (2008), independently. Mihai established a generalized Wintgen inequality for Lagrangian submanifolds in complex space forms in 2014 and he obtain the DDVV inequality, also known as generalized Wintgen inequality, for Legendrian submanifolds in Sasakian space forms in 2017. In this study, we consider generalized Wintgen inequality for Legendrian submanifolds in cosymplectic and Kenmotsu space forms.

**On strong ideals of seminearrings,** Kavitha Koppula, Babushri Srinivas Kedukodi and Syam Prasad Kuncham  Manipal Institute of Technology, India babushrisrinivas@gmail.com

We introduce the notion of strong ideal of a seminearring $S$ and give its characterization in terms of kernel of a homomorphism. If $S$ is a nearring or a semiring then every ideal of $S$ is a strong ideal. The definition of strong ideal of a seminearring has the vices of two definitions, namely, the ideal of a nearring and the ideal
of a semiring. We study homomorphisms satisfying the tame condition and prove that the quotient structure \( S_T(I) \) formed by the partitioning ideal \( I \) is isomorphic to the quotient seminearring \( S/I \). We further establish isomorphism theorems in seminearrings by using strong ideals, thus providing a generalization of respective results in nearrings and semirings.

On a conjecture by Akiyama and Pethoe concerning the distribution of polynomials with bounded roots, Peter Kirschenhofer and Joerg Thuswaldner

Peter Kirschenhofer
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About a century ago Issai Schur introduced what is now called the \( d \)-dimensional Schur-Cohn region \( \mathcal{E}_d \), defined as the set of all coefficient vectors of monic polynomials of degree \( d \) each of whose roots lies in the open unit disk. In 1922 Cohn developed an algorithm to check if a vector belongs to \( \mathcal{E}_d \), and in the following decades further properties of the Schur-Cohn region were published.

In the 1970s, in the context of discrete time systems design, Fam calculated the \( d \)-dimensional Lebesgue measure \( \lambda_d(\mathcal{E}_d) \) and proved that

\[
v_d := \lambda_d(\mathcal{E}_d) = 2^d \prod_{j=1}^{[d/2]} \left( 1 + \frac{1}{2j} \right)^{2j-d}.
\]

During the last years, \( \mathcal{E}_d \) occurred in connection with number systems and so-called shift radix systems.

If \( P \in \mathbb{R}[X] \) is a polynomial of degree \( d \) with \( r \) real and \( 2s \) nonreal roots \( (r, s) \) is called the signature of \( P \). Akiyama and Pethő studied the subsets \( \mathcal{E}_d^{(s)} \) of polynomials with fixed signature \( (d - 2s, s) \) in \( \mathcal{E}_d \) and their volumes \( v_d^{(s)} := \lambda_d(\mathcal{E}_d^{(s)}) \). They were able to establish relations of \( v_d^{(s)} \) with Selberg integrals and their generalizations by Aomoto. In particular they derived

\[
v_d^{(0)} = 2^{d(d+1)/2} \prod_{j=1}^{d} \frac{(j - 1)!^2}{(2j - 1)!}.
\]

Furthermore they proved that the numbers \( v_d^{(s)} \) are rational and that \( v_d/v_d^{(0)} \) is an odd integer for all \( d \geq 1 \). Following numerical experiments, they stated the
Conjecture. The quotient $v^{(s)}_d / v^{(0)}_d$ is an integer for all $s \leq d/2$.

In particular, for the special instance $d = 2s$ they conjectured $v^{(s)}_{2s} / v^{(0)}_{2s} = 2^{2s(s-1)}(2s/s)$. 

In the presented work we confirm the above conjecture and even prove explicit formulæ for $v^{(s)}_d / v^{(0)}_d$ for arbitrary $s \leq d/2$ and, as a by-product, verify the formula for the instance $d = 2s$ from above. The ingredients of our proofs comprise Selberg type integrals, determinants similar to the Cauchy double alternant and principal minors of partial Hilbert matrices.

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On the spectra of bipartite multidigraphs, Sasmita Barik and Gopinath Sahoo
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We define adjacency matrix as well as Laplacian matrix of a multidigraph in a new way and study the spectral properties of some bipartite multidigraphs. It is well known that a simple undirected graph is bipartite if and only if the spectrum of its adjacency matrix is symmetric about the origin (with multiplicity). We show that the result is not true in general for multidigraphs and supply a class of non-bipartite multidigraphs which have this property. We describe the complete spectrum of a multi-directed tree in terms of the spectrum of the corresponding modular tree. In case of the Laplacian matrix of a multidigraph, we obtain a necessary and sufficient condition for which the Laplacian matrix is singular. Finally, it is proved that the absolute values of the components of the eigenvectors corresponding to the second smallest eigenvalue of the Laplacian matrix of a multi-directed tree exhibit monotonicity property similar to the Fiedler vectors of an undirected tree.

Structure of Spectrum for Third Order Differential Operator with Irregular Boundary Conditions, Daulet B. Nurakhmetov, Almir A. Aniyarov and Serik A. Jumabayev daulet.kaznu@gmail.com
John Locker (2006) considered a spectral problem for the differential operator $L$ in the case of $n$-fold ($n = 2\nu \geq 2$) differentiation and with irregular boundary conditions on a finite interval.

John Locker (2006, 2008) formulated several unsolved problems of spectral analysis of two-point boundary value problems. Problem 11th is the following question: Can we find an analogous model of an odd order differential operator $L$? Are there more general models with $\sigma(L) = \mathbb{C}$ and $\rho(L) = \emptyset$, where $L$ has variable coefficients?

Serik Jumabayev and Daulet Nurakhmetov (2017) partially solved John Locker’s 11th problem in the case of a triple differentiation operator with asymmetric weight function. In this report, we will present the results for more general models.

The inverse problem for finding the right-hand side of the degenerating equation of a mixed elliptic-hyperbolic type, Kamil B. Sabitov sabitov_fmf@mail.ru

For a mixed-type equation that has important applications in gas dynamics in the theory of transonic flows of liquids and gases,

$$Lu \equiv (\text{sign } y)|y|^nu_{xx} + u_{yy} = f(x), \quad n = \text{const} > 0,$$

in a rectangular area $D = \{(x, y)|0 < x < l, -\alpha < y < \beta\}$, where $l, \alpha > 0$ and $\beta$ are given positive numbers, the inverse problem is posed: find functions $u(x, y)$ and $f(x)$ satisfying conditions: $u(x, t) \in C^1(D) \cap C^2(D_+ \cup D_-)$; $f(x) \in C(0, l) \cap L_2[0, l]$; $Lu(x, y) \equiv f(x), \quad (x, y) \in D_+ \cup D_-; u(0, y) = u(l, y) = 0, \quad -\alpha \leq y \leq \beta; u(x, \beta) = \varphi(x), \quad u(x, -\alpha) = \psi(x), \quad 0 \leq x \leq l; u_y(x, -\alpha) = g(x), \quad 0 \leq x \leq l$, where $\varphi(x), \psi(x)$ and $g(x)$ are given a sufficiently smooth function, and $\varphi(0) = \lambda(l) = 0, \psi(0) = \psi(l) = 0, g(0) = g(l) = 0, D_+ = D \cap \{y > 0\}, D_- = D \cap \{y < 0\}$.

Earlier, the inverse problem for $n = 0$ was investigated by spectral analysis. A criterion for the uniqueness of the solution of the problem was established. The functions $u(x, y)$ and $f(x)$ were constructed as the sum of a series in the system of eigenfunctions of the corresponding one-dimensional spectral problem. In justifying the convergence of series, the problem of small denominators arose.
In this paper we establish a criterion for the uniqueness of the solution of the inverse problem for all $n > 0$. The solution is constructed in explicit form as the sum of orthogonal series with the justification of convergence in classes of regular solutions under certain sufficient conditions with respect to the boundary functions $\varphi(x)$, $\psi(x)$ and $g(x)$. The stability of the solution of the problem from boundary functions with respect to the norms of spaces $L_2[0, l]$, $C[0, l]$ and $C(\bar{D})$.


Let $R$ be a right zero-symmetric near-ring with identity. We consider matrices over $R$ as defined in J.D.P. Meldrum and A.P.J. van der Walt, Matrix near-rings, *Arch. Math.* 47 (1986), 312–319. It is shown that the inverse $U^{-1}$ of a bijective matrix map $U : R^2 \to R^2$ is not necessarily a matrix map.

**Random walks on dynamical random environments with slow mixing**, Marcelo R. Hilario mhilario@mat.ufmg.br

Random motions in a random media are of major interest in mathematics and physics. They have been studied through a range of different techniques: numerical, theoretical and rigorous. Within probability theory, over the last four decades there has been much interest in the study of random walks in static random environments. More recently, many works have been devoted to understanding also the dynamic random environments. Most papers in the literature on random walk in dynamic random environment require fast or uniform space-time mixing conditions on the environment. In this talk we mention some recent developments achieved in the last few years which allowed to weaken the mixing conditions. We report on recent works where the dynamic random environment either consists of a conservative particle system (namely a Poisson field of independent simple random walks) or of interacting particle systems exhibiting a mild space-time mixing condition (more specifically a power-law type covariance decay). With the help of renormalization techniques, couplings and the construction of so-called regeneration times, we can show that the displacement of the random walk in these
environments satisfies a strong law of large numbers as well as some large deviation bounds. In some cases we are also able to prove a invariance principles.

This talk is based on joint works with O. Blondel, F. den Hollander, V. Sidoravicius, R.S. dos Santos and A. Teixeira Teixeira.

**Higher Order Differential Subordinations for Generalized Fractional Differintegral Operators,** Teodor Bulboaca, Hanaa M. Zayed, Adela O. Mostafa and Mohamed K. Aouf bulboaca@math.ubbcluj.ro

The lecture deals with some recent results obtained by the author involving the third order differential subordination basic results. Using a recent result of J. A. Antonino and S. S. Miller, that represents an extension of the technique of admissible functions introduced earlier by P. T. Mocanu and S. S. Miller, we defined and studied certain new classes of admissible functions for the case of third order differential subordinations.

The main properties we obtained allowed us to investigate some applications of the third order differential subordination for multivalent functions associated with some well-known generalized fractional differintegral operators. These new results extend a few of the previous classical ones belonging to different authors, and moreover, some interesting applications obtained for appropriate choices of the dominants were also given.

**A positive answer for a question proposed by K. Mahler,** Diego Marques UNIVERSIDADE DE BRASÍLIA, BRAZIL diego@mat.unb.br

In 1902, P. Stäckel proved the existence of a transcendental function \( f(z) \), analytic in a neighbourhood of the origin, and with the property that both \( f(z) \) and its inverse function assume, in this neighbourhood, algebraic values at all algebraic points. Based on this result, in 1976, K. Mahler raised the question of the existence of such functions which are analytic in \( \mathbb{C} \). In this lecture, we shall provide a positive answer for this question by showing the existence of uncountable many of these functions. This is a joint paper with Carlos Gustavo Moreira
Maker-Breaker fixed graph game on random graphs, Rajko Nenadov, Angelika Steger and Milos Stojakovic rajko.nenadov@monash.edu

We look at the Maker-Breaker $H$-game played on the edge set of the random graph $G_{n,p}$. In this game two players, Maker and Breaker, alternately claim unclaimed edges of $G_{n,p}$, until all the edges are claimed. Maker wins if he claims all the edges of a copy of a fixed graph $H$; Breaker wins otherwise.

We show that, with the exception of a restricted family of graphs including trees and triangles, the threshold probability for Maker’s win in this game is given by the threshold of the corresponding Ramsey property of $G_{n,p}$ with respect to the graph $H$. Asymptotically, this means that for most graphs $H$ Breaker can win the game with high probability as long as the graph does not have Ramsey property, which ensures Maker’s win by strategy stealing. To prove that, we also develop two general criterions for Breaker’s win on arbitrary graphs.

As for graphs $H$ with 2-density exactly 2, which is determined by triangle subgraphs only, we show that the threshold probability for Maker’s win can be anywhere between $9/5$ and 2.

Fredholm modular operators and representable K-theory of inverse limits of C*-algebras, Kamran Sharifi Shahrood University of Technology, Iran sharifi.kamran@gmail.com

Countable inverse limits of C*-algebras (or what are now known as $\sigma$-C*-algebras) arise naturally in the study of coarse Baum–Connes assembly map and noncommutative analogues of classical Lie groups. If $A$ is a $\sigma$-C*-algebra, we show that the inclusion map from the space of all $A$-Kasparov cycles to the space of all $A$-Fredholm operators is a weak homotopy equivalence. Then we use the representable K-theory of $\sigma$-C*-algebras and the Milnor $\lim^1$-exact sequence for RK-functor to show that the space of $A$-Fredholm operators, represents the functor $X \mapsto \operatorname{RK}_0(C(X, A))$ from the category of countably compactly generated spaces to the category of abelian groups. This fact can be regarded
as a generalization of Atiyah-Jänich Theorem for $\sigma$-$C^*$-algebras. In particular, we show that the Grothendieck group of $A$-vector bundles over $X$ need not be isomorphic to $[X, \mathcal{F}(H)]$ of homotopy classes of continuous maps from $X$ to the space of Fredholm operators on $H = l^2(A)$. We finally obtain a Milnor $\lim^1$-exact sequence for homotopy groups of $A$-Fredholm operators.

**Probabilistic Aspects on Domination Theory in Graphs**, Nader Jafari Rad

A subset $T$ of vertices in a hypergraph $H$ is a transversal if $T$ has a nonempty intersection with every edge of $H$. The transversal number of $H$ is the minimum size of a transversal in $H$. The independence number of a hypergraph $H$ is the cardinality of a largest set of vertices containing no edge of $H$. A subset $S$ of vertices of a graph $G$ is a dominating set of $G$ if every vertex in $V(G) - S$ has a neighbor in $S$. The domination number of $G$ is the minimum cardinality of a dominating set of $G$. A dominating set $S$ in a graph $G$ with no isolated vertex, is total dominating set if any vertex of $S$ is also adjacent to a vertex of $S$. The total domination number of $G$ is the minimum cardinality of a total dominating set of $G$.

We study probabilistic aspects of transversal number in hypergraphs and domination number in graphs. We first present a new probabilistic bound on the transversal number of a hypergraph, and using that obtain a new probabilistic bound on the total domination number of a graph. We thus improve the previous bounds given in [N. Alon, Transversal numbers of uniform hypergraphs, *Graphs Combin.* 6 (1990), 1–4] and [M. A. Henning and A. Yeo, A transition from total domination in graphs to transversals in hypergraphs, *Quaestiones Math.* 30 (2007), 417–436]. We next present various bounds on the domination number of a graph. We improve previous results given in [V.I. Arnautov, Estimations of the external stability number of a graph by means of the minimal degree of vertices, *Prikl. Mat. Programm.* 11 (1974), 3–8], [C. Payan, Sur le nombre
Infinite alphabet edge shift spaces via ultragraphs and their C*-algebras, Daniel Gonçalves and Danilo Royer

We define a notion of (one-sided) edge shift spaces associated to ultragraphs. In the finite case our notion coincides with the edge shift space of a graph. In general, we show that our space is metrizable and has a countable basis of clopen sets. We show that for a large class of ultragraphs the basis elements of the topology are compact. We examine shift morphisms between these shift spaces, and, for the locally compact case, show that if two (possibly infinite) ultragraphs have edge shifts that are conjugate, via a conjugacy that preserves length, then the associated ultragraph C*-algebras are isomorphic. To prove this last result we realize the relevant ultragraph C*-algebras as partial crossed products.

Singularities of differential equations in the complex plane, Thomas Kecker

A classical question in the theory of ordinary differential equations in the complex plane is to determine what types of singularities a local, analytic solution can develop when analytically continued in the complex plane. This question is addressed for various types of equations with complex analytic methods. For certain classes of second-order differential equations and two-dimensional Hamiltonian systems of equations it is shown that under certain mild conditions, the only movable singularities that can occur in a solution are of algebraic type, represented by Puiseux series with finite principle parts. In the special case that all movable singularities are poles one obtains instances of the Painlevé equations, certain integrable non-linear second-order differential equations with a rich mathematical structure, the solutions of which, the so-called Painlevé transcendents, are also re-
ferred to as non-linear special functions. A system of this type was also studied by T. Kecker (2016). One particularly interesting aspect of the Painlevé equations is the so-called Okamoto’s space of initial conditions, which is obtained by compactifying the phase space on which the original equation is defined to some rational surface, followed by a number of blow-ups, a construction to remove certain points at which the equation is of indeterminate form. In the case of the Painlevé equations, the space is uniformly foliated by the solutions of the equation and around every point in this space the equation is defined as a regular initial value problem.

The concept of the space of initial conditions is extended to certain classes of differential equations with algebraic singularities mentioned above, leading to an algorithmic procedure by which to determine, for a given equation, what different types of movable singularities can occur in the solutions of the equations in the complex plane.

**Introduction to noncommutative digital geometry,** Anna Pachol *Queen Mary University of London*

Noncommutative geometry, as the generalised notion of geometry, allows us to model the quantum gravity effects in an effective description without full knowledge of quantum gravity itself. On a curved space one must use the methods of Riemannian geometry – but in their quantum version, including quantum differentials, quantum metrics and quantum connections – constituting quantum geometry. The mathematical framework behind it is the noncommutative differential graded algebra. After presenting the motivation and the general framework, I will discuss some recent results on the classification of noncommutative differential geometries, over the finite field $\mathbb{F}_2$ (instead that of $\mathbb{C}$), in $n=2,3$ and 4 dimensions. Already in 3-dimensional cases some of the possible geometries have non zero quantum curvature, which we regard as a purely quantum phenomenon. The choice of the finite field leads to a new kind of ’discretisation scheme’, which we called the ’digital geometry’.

**Strictly positive definite multivariate covariance functions on spheres,** Jean C. Guella, Valdir A. Menegatto and Emilio Porcu *ICMC-USP and Newcastle Univer-
A function $K$ mapping a product space $Z \times Z$ into $M_p(\mathbb{C})$, is termed a \textit{matrix valued positive definite kernel} (PD) on $Z$ if
\[ \sum_{\mu, \nu=1}^{n} c_\mu K(z_\mu, z_\nu)c_\nu^* \geq 0, \]
for $n \geq 1$, distinct points $z_1, z_2, \ldots, z_n$ in $Z$ and complex vectors $c_1, c_2, \ldots, c_n \in \mathbb{C}^p$. The strict positive definiteness (SPD) of a positive definite kernel $K$ demands that the inequalities above be strict when at least one of the vectors $c_i$ is nonzero.

In 1942, I. J. Schoenberg characterized the continuous functions $f : [-1, 1] \to \mathbb{C}$ such that the kernel $(x, y) \in S^d \times S^d \to f(x \cdot y) \in \mathbb{C}$ is PD, where $\cdot$ is the usual inner product in $\mathbb{R}^{d+1}$ as having a series representation in terms of certain Gegenbauer polynomials.

The characterization for SPD in the case $d \geq 2$ appeared in 2003 in a work of Menegatto et al., and it is related on how the set of positive coefficients behave.

The aim of this talk is to give a characterization for the matrix valued version of those two theorems.

\textbf{From Golden ratio to Euler and Beyond: On practical applications of Theory of Stationary and Non-Stationary populations}, Arni S. Rao arrao@augusta.edu

The subject of population dynamics is almost one thousand years old and is been studied by famous mathematicians such as Fibonacci, d’Alambert, Daniel Bernoulli, Euler, etc. Concepts such as stability and stationarity of population are essential pillars of population dynamics. In the last century the works by Alfred Lotka laid the foundation for the population stability theory, which was developed further by William Feller through renewal equations. Ansley Coale and Norman Ryder (during 1960s and 1970s) brought several properties of stationary populations from the Life Table perspective. During last decade new identities of stationary population have emerged due to Carey’s equality (early 2000s). James Carey’s experimental work and deeper insights helped to discover newer perspectives of
stationary populations by Vaupel (2009) and Goldstein (2009). Rao and Carey (2015) have published a fundamental theorem in stationary population using insights from Carey’s Equality by blending with algebraic and combinatorial principles. Further they are building a theory which can study convergence of stationary and non-stationary populations and methods to study oscillations of population sizes. These newer results bring similar patterns that are comparable to renewal type of theory due to Lotka, Feller and others. Rao and Carey have built on the technicalities of stationary and non-stationary population theories. This talk concludes with practical applications on human population dynamics and other applications to the society. This presentation is a joint work with James R. Carey.

Relations between unification of the multidimensional Bernstein polynomials and p-adic integrals, Yilmaz Simsek ysimsek63@gmail.com

In our recent paper ”Generating functions for unification of the multidimensional Bernstein polynomials and their applications, Math Meth Appl Sci. 2018;1–12, DOI: 10.1002/mma.4746”, we constructed generating functions for m-dimensional unification of the Bernstein basis functions. By applying p-adic integral methods to the m-dimensional unification of the Bernstein basis functions and their identities, we derive many new formulas and relations including the Bernoulli numbers and polynomials, the Euler numbers and polynomials, the Stirling numbers and other special numbers and polynomials. Finally, we give further remarks and observations, and also surveys on these functions and our results.

On General Column Distance for Convolutional Codes, Marcelo Firer, Diego Napp and Sara D. Cardell Unicamp, Brazil mfirer@ime.unicamp.br

In 1991 Wei introduced the concept of generalized Hamming weights (GHW) and the Hamming weight hierarchy for linear block codes (HWH). Although the GHW were originally introduced motivated by applications in cryptography, it was Forney in 1995 that explicitly showed the close and deep connections between the HWH (also called length/dimension profile) and the trellis complexity.
The GHW have, since then, generated notable interest among coding theorists. The results obtained for block codes could be extended to convolutional codes. This motivated the work done by Rosenthal and York in 1997 where the convolutional case was addressed. In this contribution we aim at continuing this line of research by further investigating the GHW in the context of convolutional codes. Our work can be considered as an extension of the work on the GHW for free distance of convolutional codes by Rosenthal and York. In particular, we focus on column distances instead of on the free distance and propose a definition of generalized column distance (GCD) for convolutional codes. As we will see, most of the properties of the GHW for block codes apply also for this notion of column distances of convolutional codes.

Forcing of closed orbits for tight Reeb flows on $S^1 \times S^2$, Diego Alfonso Sandoval Salazar *University of Sao Paulo*

A 1–form $\lambda$ defined on a smooth $2n + 1$-dimensional manifold $M$ is called a contact form if $\lambda \wedge (d \lambda)^n$ is a form of volume on $M$. The contact form determines the tangent hyperplane distribution $\xi = \ker \lambda \subset TM$, which is called structure contact, and the vector field $R_\lambda$ defined by $\lambda(R_\lambda) = 1$ and $d\lambda(R_\lambda, \cdot) = 0$, called the Reeb field associated to $\lambda$. The pair $(M, \xi = \ker \lambda)$ is called a co-oriented contact manifold. The flow of $R_\lambda$ on $M$ is called the Reeb flow associated to the contact form $\lambda$.

Now, we suppose that $M$ has dimension 3. We say that $\xi = \ker \lambda$ is overtwisted if there exists an embedded disc $D \hookrightarrow M$ such that $T_p D = \xi_p$ for all $p \in \partial D$. Otherwise, $\xi$ is called tight. According to a classification of contact structures on $S^1 \times S^2$, there exists an unique tight contact structure up to diffeomorphism.

The main goal of this short communication is to speak the implied existence of closed orbits for Reeb flows in $S^1 \times S^2$ equipped with the tight contact structure, ie, we shown that the existence of a certain closed orbits forces the existence of infinitely many closed orbits. This result is a version of the Poincaré-Birkhoff theorem for tight Reeb flows in $S^1 \times S^2$. 
A family of contact forms on $S^1 \times S^2$ is given as follows: let

$$[-1, 1] \ni \eta \mapsto (f(\eta), g(\eta)) \in \mathbb{R}^2 \setminus \{0\},$$

be a smooth curve satisfying

(i) $f(-1) > 0, g(-1) = 0, f(1) < 0, g(1) = 0.$
(ii) $g(\eta) > 0, \forall \eta \in (-1, 1).$
(iii) $(fg' - gf')(\eta) > 0, \forall \eta \in (-1, 1).$
(iv) $(f'g'' - g'f'')(\eta) > 0, \forall \eta \in (-1, 1).$

On $S^1 \times S^2$ we have the coordinates $(t; \theta, \varphi)$ where $t \in S^1 \equiv \mathbb{R}/2\pi \mathbb{Z}$. Using these coordinates, it is possible to show that the 1-form

$$\lambda_{f,g} := f(-\cos \varphi)dt + g(-\cos \varphi)d\theta,$$

is a smooth contact form on $S^1 \times S^2$ inducing the tight contact structure. Moreover, it is possible to show that $P_1 := S^1 \times (0, 0, 1)$ and $P_2 := S^1 \times (0, 0, -1)$

are closed orbits of the Reeb flow associated to $\lambda_{f,g}$. If $f_0(\eta) = -\eta$ and $g_0(\eta) = \frac{1-f_0(\eta)^2}{2}, \eta \in [-1, 1]$ then $\xi_0 = \ker \lambda_0$ is the contact structure induced by $\lambda_0 := \lambda_{f_0,g_0}$.

**Theorem:** Let $\lambda$ be a contact form on $S^1 \times S^2$ which induces the tight contact structure $\xi_0$. Suppose that the link $L := P_1 \sqcup P_2 \subset S^1 \times S^2$ is formed by a pair of closed orbits for the Reeb flow of $\lambda$. Then it admits infinitely many other closed orbits whose homotopy classes in the complement of $L$ are prescribed according to the rotation number of its components.

The proof of this theorem makes use of the contact homology in the complement of the link $L$, an invariant constructed from the Reeb flow in the complement of a finite number of closed orbits. We show that in the case of the above theorem, this invariant is trivial for infinitely many homotopy classes in the complement of $L$. This fact implies the existence of infinite closed orbits.

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Model of volatility at closure prices of the PFCEMARGOS shares between 16 May 2013 to 31 May 2017, Christian Camilo Cortes Garcia and Fiorella
In the area of cement production, Argos is the largest cement company in Colombia and has a 51% share in the Colombian market. It is also the fourth largest cement producer in Latin America and the United States and the only producer of white cement in Colombia.

Argos has two types of shares, ordinary and preferential, that are listed on the Colombian Stock Exchange. PFCEMARGOS preferential shares began trading on May 13, 2013, compared to CEMARGOS shares, on December 26, 2005, the first being one of the most recent shares, which generates a different option for investors.

For this reason, the objective of this paper is to forecast the volatilities of the daily returns of the closing prices PFCEMARGOS, listed by the Colombian Stock Exchange, when taking as a sample the period between May 16 2013 to May 31 2017. The volatility of the daily returns is modeled by using historical volatility, with a mobile window of 20 data, and the GARCH, TGARCH, IGARCH, EGARCH and APARCH models in order to select the one that best fits the data set through the Bayesian information criterion (BIC). Subsequently, a contrast is made with the model SV$t$-AR(1) to determine the effectiveness of the models outside the sample.

A generalization of the modified self-shrinking generator, Sara D. Cardell and Amparo Fúster-Sabater sdcardell@ime.unicamp.br

In this work, we propose a generalized version of the modified self-shrinking generator introduced by Kanso in 2010 by using and extended selection rule based on the XORred value of $t$ bits of a PN-sequence. Via the concept of cyclotomic coset, we classify the generated sequences and analysed their characteristics. Emphasis is on the linear complexity of such sequences. For some values of $t$, the $t$-modified sequences produced by this new generator coincide with the sequences produced by the generalized self-shrinking generator introduced by Hu and Xiao in
2004. Thus, the $t$-modified self-shrinking generator here proposed provides one with a large class of sequences most of them with a clear application to stream cipher cryptography.

Construction of G2-instantons via twisted connected sums, Henrique N. Sá Earp henrique.saearp@ime.unicamp.br

G2-instantons are particular solutions to the Yang-Mills equation over certain 7-manifolds with special holonomy contained in the Lie group G2, called G2-manifolds. An important technique to obtain compact G2-manifolds is called the twisted connected sum construction, which was suggested by Donaldson and developed by Kovalev and Corti-Haskins-Nordström-Pacini. It consists in gluing two suitable asymptotically cylindrical Calabi-Yau (ACylCY) 3-folds along their $K3$ asymptotic cross-sections, together with a ‘twist’ by an addition real $S^1$-component.

We will describe a method to produce G2-instantons over a compact twisted connected sum G2-manifold, applying a previous gluing theorem by T. Walpuski and the author to a pair of instantons over noncompact G2–manifolds with a tubular end. Moreover, the moduli spaces of the ingredient instantons are non-trivial, and their images in the moduli space over the asymptotic cross-section $K3$ surface intersect transversely.

More precisely, such tubular instantons correspond to so-called asymptotically stable holomorphic vector bundles over the matching ACylCY pair. These are obtained using a twisted version of the Hartshorne-Serre correspondence between bundles of rank $r$ and subschemes of codimension $r$, which can be used to produce many more examples. Moreover, their deformation theory and asymptotic behaviour are explicitly understood, results which may be of independent interest.

This talk is based on joint work with G. Menet and J. Nordström, and it builds upon previous works with T. Walpuski and also M. Jardim, G. Menet and D. Prata.

Numerical approximation of the Stokes eigenvalue problem in cubic domain based on penalty technique, Ramon Codina and Önder Türk ramon.codina@
We consider numerical solution strategies for the Stokes eigenvalue problem based on the use of penalty formulations. There are several difficulties in approximating accurately the eigenmodes of the Stokes problem in a cubical domain such as the lack of theoretical knowledge of the corresponding spectrum, and algorithmic limitations due to the size of the resulting algebraic system. These difficulties are emphasised when the problem is formulated in primitive variables, velocity and pressure, as the discretisation of the operator leads to a generalised eigenvalue problem with zero diagonal entries in the resulting algebraic system. In this study, we exploit the well known benefits of penalisation approach in incompressible Navier-Stokes models to cure the associated pathologies. We show that the classical penalty method can successfully be adapted for the eigenproblem. Moreover, we propose an iterative approach that allows relatively large penalisation parameters and leads to an algebraic system with better conditioning. The implementation of this idea results in a modified eigenproblem known as inhomogeneous generalised eigenvalue problem which requires special attention compared to the standard ones. We investigate several solution procedures based on Newton’s method. We use the Chebyshev spectral collocation method based on expanding the unknown fields in tensor product of Chebyshev polynomials for the spatial discretisation, even though the idea applies to any discretisation technique. We show that the proposed schemes are computationally cheap and efficient, and the numerical results for two-dimensional (square) and three-dimensional (cube) problems are in good agreement with the published ones.

**On weighted Dirichlet spaces of quaternion-valued functions,** Karen L. Avetisyan and Klaus Guerlebeck avetkaren@ysu.am

In a recent paper of ours [J. Morais, K. Avetisyan, K. Gürlebeck, On Riesz systems of harmonic conjugates in \( \mathbb{R}^3 \), Math. Methods Applied Sci. 2013, vol. 36, No. 12, 1598–1614], we proved that a ”harmonic conjugation” operator constructed by special systems of homogeneous harmonic and monogenic polynomials is not bounded in weighted Bergman spaces \( L^2_\alpha(B_3) \) of reduced quaternion-valued functions in the unit ball \( B_3 \subset \mathbb{R}^3 \).
In this talk, we prove that, in contrast to the Bergman spaces case, the same operator is bounded in weighted Dirichlet spaces $\mathcal{D}_\alpha^2(B_3)$ of quaternion-valued functions in the ball $B_3$. Furthermore, applying another approach for a construction of harmonic conjugates, we sharpen and extend the result to weighted Dirichlet spaces $\mathcal{D}_\alpha^p(B_3)$, $1 < p < \infty$, $\alpha > -1$, of monogenic functions,

$$\|f\|_{\mathcal{D}_\alpha^p} := \|\overline{D}f\|_{L_p} = \left(\int_{B_3} |\overline{D}f(x)|^p (1 - |x|^2)^\alpha \, dV(x)\right)^{1/p} < +\infty,$$

where $\overline{D}f = \frac{\partial f}{\partial x_0} - i\frac{\partial f}{\partial x_1} - j\frac{\partial f}{\partial x_2}$ is the conjugate generalized Cauchy-Riemann operator on $\mathbb{R}^3$, and $dV(x)$ stands for the normalized Lebesgue volume measure on $B_3$.

**Estimate exponential memory decay in hidden Markov model and its applications**, Felix X. Ye, Yian Ma and Hong Qian yexf308@uw.edu

Inference in hidden Markov model has been challenging in terms of scalability due to dependencies in the observation data. In this paper, we utilize the inherent memory decay in hidden Markov models, such that the forward and backward probabilities can be carried out with subsequences, enabling efficient inference over long sequences of observations. We formulate this forward filtering process in the setting of the random dynamical system and there exist Lyapunov exponents in the i.i.d random matrices production. And the rate of the memory decay is known as $\lambda_2 - \lambda_1$, the gap of the top two Lyapunov exponents almost surely. An efficient and accurate algorithm is proposed to numerically estimate the gap after the soft-max parametrization. The length of subsequences $B$ given the controlled error $\epsilon$ is $B = \log(\epsilon)/(|\lambda_2 - \lambda_1|)$. We theoretically prove the validity of the algorithm and demonstrate the effectiveness with numerical examples. The method developed here can be applied to widely used algorithms, such as mini-batch stochastic gradient method. Moreover, the continuity of Lyapunov spectrum ensures the estimated $B$ could be reused for the nearby parameter during the inference.

**Design of geometrically uniform signal sets via Whittaker surfaces**, Erika P.
The geometric uniformity of a signal set is a property associated with its isometry group. Thus, the extension of this concept to the hyperbolic plane implies knowing the hyperbolic isometries as well as its geometric properties. A signal set \( S \) is said to be geometrically uniform (GU) if for each \( x, y \in S \), its orbit, by the isometry group \( \Gamma(S) \), is all of \( S \). Thus, it follows that if \( S \) is a GU signal set and for any \( x, y \in S \) then there exists an isometry \( u \in \Gamma(S) \) such that \( u(x) = y \) and that the Dirichlet regions are all congruent, i.e., \( u(R_V(x)) = R_V(y) \). There exists a matched labeling between \( S \) and a group \( G \) if, and only if, \( G \) is isomorphic to a transitive subgroup of \( \Gamma(S) \). We propose a new approach to the problem of designing a traditional digital communication system. The steps to achieve this goal are as follows: 1) let \( C \) be a given discrete memoryless channel; 2) from the 2-cell embedding of \( C \), find the minimum and maximum genus of the associate oriented compact surface; 3) establish the hyperelliptic curve, \( y^2 = f(z) = (z-s_1)(z-s_2)\ldots(z-s_{2g+2}) \), associated with each genus found in 2) by selecting a set of \( 2g+2 \) symmetric points \( S = \{s_1, s_2, \ldots, s_{2g+2}\} \); 4) find the corresponding monodromy group of the second order Fuchsian differential equation \( \ddot{y} + W(X^{-1}(z))y(z) = 0 \), where \( W(.) \) denotes the Schwarzian derivative of the multivalued function \( X^{-1}(z) \), the inverse mapping from \( C \cup \{\infty\} \rightarrow \mathbb{D} \), where \( \mathbb{C} \) denotes complex numbers and \( \mathbb{D} \) the Poincaré disk, such that the link between the algebraic curve and the group is assumed valid, and so \( W(X^{-1}(z)) \) is known in terms of the parameters \( s_1, s_2, \ldots, s_{2g+2} \) as conjectured by Whittaker; 5) find the generators of the group whose fundamental (Dirichlet) region uniformizes the given algebraic curve; 6) find the matched labeling between the signal set \( S \) and the algebraic structure associated with the alphabet of the algebraic-geometric code. We consider the cases where the algebraic curves (Whittaker surfaces) are of the form as shown by Rankin, \( y^m = z^p(z^n-1)^r \), leading to the \( 2g+2 \) symmetric points \( s_1, s_2, \ldots, s_{2g+2} \). Consequently, the designed signal set is geometrically uniform.

Isotopies of discrete cotangent flows in the presence of a Lie group of symmetries with perfect Lie algebra, Nicolás Borda and Javier Fernandez and Marcela Zuccalli nborda@mate.unlp.edu.ar
The discretization of time in Lagrangian mechanics leads to a notion of discrete mechanical system with flows in both a space of pairs of positions, \( Q \times Q \), and via a discrete analogue of the Legendre transform, in \( T^*Q \). The so-called discrete cotangent flows behave, in some ways, as —continuous-time— Hamiltonian ones; for example, they preserve the canonical symplectic form, and the Liouville volume, as well as the standard moment map when there exist symmetries. All of the previous properties are relevant for numerical applications.

For many discretizations of Lagrangian systems in practice, the resulting discrete cotangent flows provide a smooth —symplectic— isotopy starting at the identity of \( T^*Q \), that is, when the discretization parameter or time step \( h \) is 0. It is a known fact that, locally, this family of flows corresponds to an \( h \)-time-dependent Hamiltonian vector field, thus inheriting any invariant of its class. In this work we firstly detail the existence of such an isotopy, and then show that, for every \( h \), the Hamiltonian function in question can be chosen invariant in the presence of a connected Lie group of symmetries with perfect Lie algebra. To this end, we prove an equivariant version of the Poincaré lemma for closed 1-forms under the action of a group of that type. All in all, we obtain sufficient conditions to extend any common invariant among Hamiltonian systems, now, with symmetries, to their discrete counterpart —such as the standard moment map—.

A generalized formulation of heat and mass transfer in domains with cuts, Dmytro Nomirovskii  
Dmytro Nomirovskii  kashpir74@gmail.com

There are many physical processes occurring in media with foreign insertions. Inside the domain or on its boundary, there can be thin liquid layers and gas gaps, refractories, oxides and other films, cracks and cuts, various membranes, etc. When these problems are studied, the foreign part is usually eliminated from the domain, and boundary (transmission) conditions on the surfaces of the insertions are established. Thus, there is a boundary-value problem in a domain with cuts. In this way, heat and mass transfer has been investigated in many publications.

There is another approach. Instead of one second-order parabolic equation describing the dynamics of the process, a set of two first-order differential equations in natural variables can be considered, where the new variable is the vector
of flow of the substance being transported. Under this approach, the eliminated insertions are returned into the domain, and the transmission conditions are taken into account through coefficients of this set of equations.

This formulation has some advantages in comparison with the traditional ones. The new equations can be interpreted, in a generalized way though, as physical laws. The first of these equations, the scalar one, expresses the substance conservation law, and the second, the vector one, expresses the phenomenological law of transportation. Besides, the presence of several equations gives more possibilities to prove necessary properties of operators than traditional formulations do. In addition, the domain becomes simply connected that is of considerable importance in many cases, such as numerical procedures. A typical negative feature is that coefficients of the equations become generalized functions.

Based on this approach, we will construct a uniform mathematical model of heat and mass transfer, which unifies different transmission conditions, in particular, ideal contact, imperfect contact, and proper lumped source.

**First integrals for the geodesic flow on solvmanifolds**, Gabriela P. Ovando
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The aim is the study of the geodesic flow on solvmanifolds when the metric is induced by a left-invariant metric on a solvable Lie group. The idea is to relate the first integrals to the algebraic or geometric structure underlying the manifold. We shall see several examples, where Killing vectors and invariant functions are described on the Lie group.

**Gelfand-Tsetlin Theory for Rational Galois Algebras**, Pablo M. Zadunaisky, Vyacheslav Futorny, Dimitar Grantcharov and Luis E. Ramírez
*Universidade de São Paulo, Brasil and University of Texas at Arlington, USA and Universidade Federal do ABC, Brasil pzadunaisky@gmail.com*

A class of Galois algebras (rings and orders) was introduced to combine the successful representation theories of generalized Weyl algebras, and the universal
enveloping algebra of $\mathfrak{gl}_n$. The structure of these algebras intrinsically determined by a certain commutative subalgebra $\Gamma$. The class of Galois algebras is a non-commutative generalization of a classical notion of $\Gamma$-order in skew group rings. Classical example of a Galois order is the universal enveloping algebra of $\mathfrak{gl}_n$ over the Gelfand-Tsetlin subalgebra. Other well known examples include: generalized Weyl algebras over integral domains such as the $n$-th Weyl algebra, the quantum plane, the $q$-deformed Heisenberg algebra, quantized Weyl algebras, the Witten-Woronowicz algebra, finite $W$-algebras of type $A$ and, more generally, principal Galois orders.

**On the Caginalp phase-field system based on type III with two temperatures and nonlinear coupling**, Armel Andami Ovono Masuku University, Gabon

We study a generalization of the Caginalp phase-field system based on the theory of type III thermomechanics with two temperatures for the heat conduction with a nonlinear coupling term. This case is shown to be more relevant for non-simple materials. We start our analysis by establishing existence, uniqueness and regularity of the solutions. Then, we discuss dissipativity and existence of global attractors. We finish our analysis by studying the spatial behavior of the solutions in a semi-infinite cylinder, assuming the existence of such solutions.

**Fluid-structure interaction system with Coulomb’s friction law**, Loredana Bălilescu Federal University of Santa Catarina, Brasil and University of Pitești, Romania

We propose a new model in a fluid–structure interaction system composed by a rigid body and a viscous incompressible fluid using a boundary condition governed by the Coulomb’s friction law. With this boundary condition, the fluid can slip on the boundary if the tangential component of the stress tensor is too large. The governing equations are the Navier-Stokes system for the fluid and the Newton laws for the body. The corresponding coupled system can be writ-
ten as a variational inequality. We prove that there exists a weak solution of this system.

**Trunk and representativity of knots**, Makoto Ozawa and Ryan Blair *Department of Natural Sciences, Faculty of Arts and Sciences, Komazawa University, 1-23-1 Komazawa, Setagaya-ku, Tokyo, 154-8525, Japan* \texttt{w3c@komazawa-u.ac.jp}

In this talk, we investigate two geometrical invariants of knots, the trunk and the representativity.

The trunk of a smooth embedding of a knot type, which is in a Morse position with respect to the standard height function of the 3-sphere, is defined as the maximal number of the intersection between the embedding and level spheres. Then, the trunk of a knot type is defined as the minimal trunk over all embeddings in a Morse position.

The representativity of a pair of a smooth embedding of a knot type and a closed surface containing the embedding is defined as the minimal number of the intersection between all compressing disks for the closed surface and the embedding. Then, the representativity of a knot type is defined as the maximal representativity over all closed surfaces containing the embedding.

We show that for any knot, the representativity is bounded above by a half of the trunk. We also define the trunk of a tangle and show that if a knot has an essential tangle decomposition, then the representativity is bounded above by half of the trunk of either of the two tangles.

This is a joint work with Ryan Blair.

**Para-Sasakian Manifolds and *-Ricci Solitons**, Prakasha D. G. and Veeresha Pundikala \texttt{prakashadg@gmail.com}

In this paper we study a special type of metric called *-*Ricci soliton on para-Sasakian manifold $M$. We prove that if the para-Sasakian metric is a *-*Ricci soliton on a manifold $M$, then $M$ is either $\mathbb{D}$-homothetic to an Einstein manifold, or the Ricci tensor of $M$ with respect to the canonical paracontact connection vanishes.
The Intermediate Value Theorem for polynomials of low degree and the Fundamental Theorem of Algebra, Daniel Perrucci and Marie-Françoise Roy

There are many well-known proofs of the Fundamental Theorem of Algebra. Many algebraic proofs lie on the fact that the Intermediate Value Theorem holds on $\mathbb{R}$, and actually, in order to prove that a polynomial of degree $d$ in $\mathbb{C}[X]$ has a root in $\mathbb{C}$, they apply the Intermediate Value Theorem to polynomials in $\mathbb{R}[X]$ of degree exponential in $d$.

Recently, Michael Eisermann proposed a new proof for the Fundamental Theorem of Algebra, of a mixed algebraic-geometric nature, which is based on Cauchy index, winding number and Sturm chains. It also lies on the fact that the Intermediate Value Theorem holds on $\mathbb{R}$.

In this presentation, we show how to adapt Eisermann’s proof through the use of subresultants polynomials, so that given any ordered field $(\mathbb{R}, \leq)$ and $\mathbb{C} = \mathbb{R}[i]$, in order to prove that a polynomial of degree $d$ in $\mathbb{C}[X]$ has a root in $\mathbb{C}$, the Intermediate Value Theorem on $\mathbb{R}$ is only required to hold for polynomials in $\mathbb{R}[X]$ of degree bounded by $d^2$.

Nef vector bundles on a projective space with first Chern class three, Masahiro Ohno

We classify nef vector bundles on a projective space with first Chern class three over an algebraically closed field of characteristic zero; we see, in particular, that these nef vector bundles are globally generated if the second Chern class is less than eight, and that there exist nef but non globally generated vector bundles with second Chern class eight and nine on a projective plane.

Robust Modifications of U-statistics and Applications to Covariance Estimation Problems, Stanislav Minsker and Xiaohan Wei

Robust Modifications of U-statistics and Applications to Covariance Estimation Problems, Stanislav Minsker and Xiaohan Wei
Let $Y$ be a $d$-dimensional random vector with unknown mean $\mu$ and covariance matrix $\Sigma$. This work is motivated by the problem of designing an estimator of $\Sigma$ that admits tight deviation bounds in the operator norm under minimal assumptions on the underlying distribution, such as existence of only 4th moments of the coordinates of $Y$. To address this problem, we propose robust modifications of the operator-valued U-statistics based on the ideas pioneered by P. Huber, obtain non-asymptotic guarantees for their performance, and demonstrate the implications of these results for the covariance estimation problem under various structural assumptions.

**Blocking sets in $PG(2, q)$ with respect to a conic**, Bikramaditya Sahu, K. L. Patra and B. K. Sahoo NISER, Bhubaneswar, India bikram.sahu@niser.ac.in

Let $q$ be a prime power and $PG(2, q)$ be the Desarguesian projective plane over a finite field of order $q$. We denote by $\mathbb{P}$ and $\mathbb{L}$ the point set and the line set of $PG(2, q)$, respectively. For a subset $L$ of $\mathbb{L}$, an $L$-blocking set in $PG(2, q)$ is a subset $B$ of $\mathbb{P}$ such that every line in $L$ contains at least one point of $B$. Let $C$ be an irreducible conic in $PG(2, q)$. Let $E$ (respectively, $T, S$) denote the set of all lines of $PG(2, q)$ which are external (respectively, tangent, secant) to $C$. If $L = E \cup T \cup S$ and $B$ is an $L$-blocking set in $PG(2, q)$, then $|B| \geq \frac{q^2 - 1}{q - 1}$ and equality holds if and only if $B$ is a line of $PG(2, q)$. One can ask the following combinatorial question: For $L \in \{E, T, S, E \cup T, E \cup S, T \cup S\}$, find the minimum size of an $L$-blocking set and describe all $L$-blocking sets of that cardinality.

We note that this question was answered for the line sets $L$, where $L \in \{E, S, E \cup T, T \cup S\}$. In this talk, we shall discuss the minimum size $L$-blocking sets for the remaining line sets, that is, for $L \in \{T, E \cup S\}$.

**Stability analysis of Double Diffusive Convection in a Porous Medium Saturated with Power Law Fluid with Horizontal Through-Flow Considering Viscous Dissipation**, Seema Kumari and P. V. S. N Murthy seemakumari151@gmail.com
The double diffusive convective instability of horizontal throughflow of a non-Newtonian power law fluid in a horizontal porous medium is investigated by considering viscous dissipation in the medium. The lower wall of the channel is with constant heat flux while the upper wall is maintained at constant temperature. The basic steady state solution of the governing equations is determined for a horizontal throughflow. The basic temperature profile is affected by the presence of viscous dissipation term in energy equation. The critical value of Rayleigh number and the corresponding wave number are affected by the values of Péclet number ($Pe$), power law index ($n$), buoyancy ratio ($N$), Lewis number ($Le$) and Gebhart number ($Ge$). The eigenvalue problem is solved using bvp4c routine in Matlab and also using the shooting method coupled with R-K method in Mathematica. It has been noticed that the effect of viscous dissipation has increased the thermal instability of power law fluid flow. The convective instability of base flow is discussed for the longitudinal rolls and transverse rolls. It is observed that the longitudinal rolls are the most unstable mode for dilatant fluid ($n > 1$) whereas for pseudoplastic fluid ($n < 1$) the transverse rolls are the most unstable modes.

Double-Diffusive Convective Instability in a Darcy-Brinkman Porous Layer with a Horizontal Throughflow and Viscous Dissipation, Rashmi Dubey and P. V. S. N. Murthy rashmirkd2012@gmail.com

The onset of double-diffusive convective instability induced by viscous dissipation in a fluid-saturated porous layer of high permeability is investigated. The porous layer is infinitely long along the horizontal direction and is bounded by two rigid surfaces maintained at constant, but different solute concentrations. The lower surface is thermally insulated, whereas, the upper surface is considered to be isothermal. There is a horizontal throughflow in the medium. Darcy-Brinkman model is adopted for deriving the equations governing the flow in the medium. The instability in the base flow is considered to be induced by the non-negligible viscous heating. Disturbances in the base flow are assumed in the form of oblique rolls, where the longitudinal and the transverse rolls are at the two extreme inclinations. The disturbance functions are assumed to be of $O(1)$. It is considered that $Ge \ll 1$ and $|Pe| \gg 1$ and the eigenvalue, that is, $R = GePe^2$ is of $O(1)$. The eigenvalue problem with coupled ordinary differential equations governing
the disturbance in the flow is solved numerically using Matlab. Results obtained depict that the flow is most stable in the Brinkman regime and the longitudinal rolls are the preferred mode of instabilities. Solute concentration gradient has a linear monotonic impact on the instability of the flow in the medium.

Scattering of water waves by an inclined thin plate in a two-layer fluid of finite depth, Najnin Islam najnin.islam92@gmail.com

The problem of water wave scattering by an inclined thin plate submerged in the lower layer of a two-layer fluid is investigated here using linear theory. In a two-layer fluid, for a prescribed frequency, incident waves propagate with two different wavenumbers. Thus we determine the reflection and the transmission coefficients and the hydrodynamic force for both the wavenumbers. This leads to two separate problems. These problems are reduced to hypersingular integral equations for the potential difference across the plate which are solved by an expansion-collocation method. Several numerical results are presented graphically in a number of figures for the physical quantities varying the inclination, depth, and length of the plate. Making suitable adjustment of the parameters published results for a vertical plate submerged in a single layer fluid are recovered. In addition, the energy identities are used as a partial check on the correctness of the numerical results.

A combinatorial model of the free loop fibration, Manuel Rivera University of Miami, USA and Tbilisi State University, Georgia

For the free loop fibration $\Omega Y \to \Lambda Y \to Y$ on the geometric realization $Y = |X|$ of a path-connected simplicial set $X$ we construct a cellular model $|\Box X| \to |\Box X| \to |X|$, where $\Box X$ is a monoidal cubical set and $\Box X$ is an abstract set with standard cubes and freehedra as modeling polytopes. The cellular chain complex $C_*(\widehat{\Box} X)$ is identified with the standard coHochschild chain complex of the chain coalgebra $C_*(X)$ for a simply connected $X$, while for $X$ with the non-trivial fundamental group $\pi_1(X)$ generalizes it.
Forward-Backward-Half Forward Splitting for solving monotone inclusions, Luis M. Briceño-Arias and Damek Davis luis.briceno@usm.cl

Tseng’s algorithm finds a zero of the sum of a maximally monotone operator and a monotone continuous operator by evaluating the latter twice per iteration. In this paper, we modify Tseng’s algorithm for finding a zero of the sum of three operators, where we add a cocoercive operator to the inclusion. Since the sum of a cocoercive and a monotone-Lipschitz operator is monotone and Lipschitz, we could use Tseng’s method for solving this problem, but implementing both operators twice per iteration and without taking into advantage the cocoercivity property of one operator. Instead, in our approach, although the continuous monotone operator must still be evaluated twice, we exploit the cocoercivity of one operator by evaluating it only once per iteration. Moreover, when the cocoercive or continuous-monotone operators are zero it reduces to Tseng’s or forward-backward splittings, respectively, unifying in this way both algorithms. In addition, we provide a preconditioned version of the proposed method including non self-adjoint linear operators in the computation of resolvents and the single-valued operators involved. This approach allows us to also extend previous variable metric versions of Tseng’s and forward-backward methods and simplify their conditions on the underlying metrics. We also exploit the case when non self-adjoint linear operators are triangular by blocks in the primal-dual product space for solving primal-dual composite monotone inclusions, obtaining Gauss-Seidel type algorithms which generalize several primal-dual methods available in the literature. Finally we explore applications to the obstacle problem, Empirical Risk Minimization, distributed optimization and nonlinear programming and we illustrate the performance of the method via some numerical simulations.

Compact CMC Graphs in $M \times R$ with Boundary in Two Horizontal Slices, A. Aiolfi, G. Nunes, L. Sauer and R. Soares

We establish existence and uniqueness of compact graphs of constant mean curvature in $M \times R$ with boundary lying in two horizontal slices of $M \times R$, where $M$ is a complete $n$-dimensional Riemannian manifold, $n \geq 2$. Given bounded $C^{2,\alpha}$-domains $\Lambda, \Lambda_i, i = 1, ..., m$ of $M$, with $\overline{\Lambda_i} \subset \Lambda, \overline{\Lambda_i} \cap \overline{\Lambda_j} = \emptyset$ if $i \neq j$ and
such that
\[ \Omega = \Lambda \setminus \left( \bigcup_{i=1}^{m} \overline{\Lambda}_i \right) \]
is a domain and given \( h, H \geq 0 \), we investigate the Dirichlet problem
\[
\begin{cases}
Q_H(u) := \text{div} \left( \frac{\nabla u}{\sqrt{1+|\nabla u|^2}} \right) + n H = 0 \text{ in } \Omega, \ u \in C^{2,\alpha}(\overline{\Omega}) \\
u|_{\partial \Lambda} = 0, u|_{\partial \Lambda_i} = h, i = 1, \ldots, m
\end{cases}
\tag{7}
\]
where \( \text{div} \) and \( \nabla \) are the divergence and the gradient in \( M \). We provide explicit upper bounds for \( h \) for which (7) is solvable, depending on the domain data and on the geometry of \( M \), as the mean curvature of \( \partial \Omega \) and the Ricci curvature of \( M \).

**Inflection points on hyperbolic tori in the three dimensional sphere**, Ronaldo A. Garcia ronaldoalvesgarcia51@gmail.com

Families of hyperbolic tori in \( S^3 \) (the asymptotic lines are globally defined) without double inflection points is provided. More precisely, a small deformation of the Clifford torus parametrized by asymptotic lines is analyzed and it is described the set of inflections of the two families of asymptotic lines \( \mathcal{A}_1 \) and \( \mathcal{A}_2 \). Denote by \( I_i \) the set of inflections of the asymptotic lines of the associated asymptotic foliation \( \mathcal{A}_i \), also called flecnodal set. The intersection \( I_1 \cap I_2 \) is called the set of double inflections. It is shown that by an appropriated deformation of the Clifford torus the set \( I_1 \cap I_2 \) is empty for the deformed surface. This gives a negative answer to a problem formulated by S. Tabachnikov and V. Ovsienko [Hyperbolic Carathéodory Conjecture, Proc. of the Steklo Inst. of Math. 258 (2007), p. 178-193] in the context of spherical surfaces. Also, a family of immersed tori in \( S^3 \), without double inflection set, is obtained.

Joint work with Bruno Freitas (IME/UFG), work accepted in Quarterly Journal of Mathematics.

**Stabilization of dispersion-generalized Benjamin-Ono**, Seungly Oh, Cynthia Flores and Derek L. Smith seungly.oh@gmail.com

We examine \( L^2 \) well-posedness and stabilization property of the dispersion-
generalized Benjamin-Ono equation with periodic boundary conditions. The main ingredient of our proof is a development of dissipation-normalized Bourgain space, which gains smoothing properties simultaneously from dissipation and dispersion within the equation. We will establish a bilinear estimate for the derivative nonlinearity using this space and prove the linear observability inequality leading to small-data stabilization.

Multi-dimensional Fractional Transforms and Convolution Theorems, Ahmed I. Zayed DePaul University, USA azayed@depaul.edu

Let \( \mathcal{F} \) denote an integral transformation acting on a space of functions \( \mathcal{H} \). By a convolution operation, denoted by \( \ast \), associated with the transformation \( \mathcal{F} \), we mean an operation defined on \( \mathcal{H} \) such that the equation

\[
\mathcal{F} (f \ast g) = \mathcal{F}(f) \mathcal{F}(g),
\]

holds for all \( f, g \in \mathcal{H} \).

Convolution theorems for some integral transforms, such as Fourier, Laplace, and Hankel transforms, are well known and have been studied extensively. In recent years fractional integral transforms, such as fractional Fourier, Laplace, Hankel, wavelets, and Radon transforms have been developed and they have shown promising results in many engineering and physical applications. However, convolution theorems for these transforms have not been fully developed, especially for multidimensional transforms that are not tensor product of one dimensional transforms.

The purpose of this talk is to investigate different convolution theorems for some fractional integral transforms in one and several variables.

Solving the nonlinear Fredholm integral equations by Lagrange's method of finite increments, Zhyldyz Asanova, Akylbek Kerimbekov and Alymbek Uraliev zhyldyzasanova73@mail.ru

First Results on the study of nonlinear integral equations appeared at the beginning of the last century. The papers of A.Liapunoff (1906), E. Schmidt (1908),
P. Levy (1910), G. Bratu (1910), G. Buch (1912), A. Gollet (1912), H. Gagajkian (1912) were the prerequisite for the development of the nonlinear integral equations theory. Following these papers new researches of A. Hammerschtein, L. Lichtenstein, P. Uryson, R. Iglisch, V. Nemyshky appeared on this direction. The theory of nonlinear integral equations was formulated in the 1930s. Since then, interest to nonlinear integral equations hasn’t weakened. New applied problems are being appeared lead to the study of nonlinear integral equations, and new methods are being developed for solving nonlinear integral equations and for qualitative research. For example, Hammerstein investigated the nonlinear integral equation.

\[ \varphi(t) = \int_a^b K(t, s) F(s, \varphi(s)) ds, \]

when kernel \( K(t, s) \) has certain properties of the Fredholm kernel, and Uryson investigated the nonlinear integral equation

\[ \varphi(t) = \int_a^b K(t, s, \varphi(s), \lambda) ds \]

on the assumption that it has a solution \( \varphi_0(t) \) for \( \lambda = \lambda_0 \) and one isn’t a characteristic addition of the kernel \( K_\varphi(t, s, \varphi_0(s), \lambda_0) \).

In this paper, we suggest a new method to solving nonlinear Fredholm integral equations make it possible to find a solution in the form of the finite sum. Sufficient conditions are found for the unique solvability of the nonlinear Fredholm integral equation

\[ \varphi(t) = \lambda \int_a^b K(t, x, \varphi(t)) dt + f(x). \quad (1) \]

Global Results on Control and Stabilization of Fourth Order Nonlinear Schrödinger Equation on the Torus, Roberto Capistrano–Filho Universidade Federal de Pernambuco

In this talk we present some results of controllability and stabilizability of a class of distributed parameter control system described by the fourth order nonlin-
ear Schrödinger on the torus $\mathbb{T}$ with internal control acting on a sub-domain $\omega$ of $T$. More precisely, by certain properties of propagation of compactness and regularity in Bourgain spaces for the solutions of the associated linear system, we will show that the system is globally exactly controllable and globally exponentially stabilizable.

This is a join work with Márcio Cavalcante of Federal University of Alagoas (UFAL).

**Option pricing with fractional stochastic volatility and discontinuous payoff function of polynomial growth, Yuliya Mishura**

We consider the pricing problem related to payoffs of polynomial growth that can have discontinuities of the 1st kind. The asset price dynamic is modeled within the Black-Scholes framework characterized by a stochastic volatility term driven by a fractional Ornstein-Uhlenbeck process. In order to solve the aforementioned problem, we consider three approaches. The first one consists in a suitable transformation of the initial value of the asset price, in order to eliminate possible discontinuities. Then we discretize both the Wiener process and the fractional Brownian motion and estimate the rate of convergence of the related discretized price to its real value whose closed-form analytical expression is usually difficult to obtain. The second approach consists in considering the conditional expectation with respect to the entire trajectory of the fractional Brownian motion (fBm). Here we derive a presentation for the option price which involves only an integral functional depending on the fBm trajectory, and then discretize the fBm and estimate the rate of convergence of the associated numerical scheme. In both cases the rate of convergence is the same and equals $n^{-rH}$, where $n$ is the partition size, $H$ is the Hurst index of the fBm, and $r$ is the Hölder exponent of the volatility function. The third method consists in calculating the density of the integral functional depending on the trajectory of the fBm via Malliavin calculus and providing the option price in terms of the associated probability density. We propose also other models of the rough stochastic volatility. This is a joint work with Viktor Bezborodov and Luca Di Persio.
Topology of the punctual Hilbert schemes for the curve singularity of type $A_{2d}$, Masahiro Watari watari@jadypm.edu.my

We describe Euler numbers and Betti numbers of the punctual Hilbert schemes for the curve singularity of type $A_{2d}$. Let the notation $[\cdot]$ be the Gauss’ symbol. Namely, for a real number $a$, the value $[a]$ is the greatest integer that is less than or equals to $a$. The followings are our main theorems:

**Theorem 1** For two integers $d$ and $r$ with $1 \leq r \leq 2d$, let $\mathcal{M}_{d,r}$ be the punctual Hilbert scheme of degree $r$ for the curve singularity of type $A_{2d}$. The Euler number of $\mathcal{M}_{d,r}$ equals $\left\lfloor \frac{r}{2} \right\rfloor + 1$.

**Theorem 2** Let $r$ be a positive integer. The odd (co-)homology groups of the punctual Hilbert scheme $\mathcal{M}_{d,r}$ for the curve singularity of type $A_{2d}$ are zero. On the other hand, its even (co-)homology groups are free abelian groups with the Betti numbers

$$h_{2m}(\mathcal{M}_{d,r}) = h^{2m}(\mathcal{M}_{d,r}) = \begin{cases} 1 & \text{if } m \in \{0, 1, \ldots, \left\lfloor \frac{r}{2} \right\rfloor\}, \\ 0 & \text{if } m \notin \{0, 1, \ldots, \left\lfloor \frac{r}{2} \right\rfloor\}. \end{cases}$$

Finding conditions to control the spread of dengue via analysis of invariant manifolds of a 4-dimensional dynamical system, Dana Contreras and Pablo Aguirre dana.cjulio@gmail.com

The introduction of *Wolbachia* bacteria into *Aedes aegypti* mosquito population is one of the most promising tools for controlling the spread of arboviruses such as dengue, zika and chikungunya. *Wolbachia* is transmitted by the female mosquito to her offspring and dramatically reduces the mosquito’s ability to acquire and transmit arbovirus infections.

Recently, Campo-Duarte et al. presented a population dynamics model for interaction between wild female mosquitoes and those infected with *wMelPop Wolbachia* strain. In this model, both types of mosquitoes compete for the same vital resources and share the same locality. In this work, we extend this model by adding a one-dimensional spatial propagation with diffusion in a bounded domain.
Thus we consider the following system of partial differential equations:

\[
\begin{align*}
\frac{\partial F}{\partial t} &= \left( \Psi_f - \frac{\Psi_f - \delta_f}{K_f} (F + W) \right) F \left( \frac{F}{K_0} - 1 \right) - \delta_f F + D_F \frac{\partial^2 F}{\partial x^2}, \\
\frac{\partial W}{\partial t} &= \frac{W}{K_W} (K_W - F - W) r_W + D_W \frac{\partial^2 W}{\partial x^2}.
\end{align*}
\] (8)

In the equation above, $F \geq 0$ denotes healthy females and $W \geq 0$ denotes females infected with Wolbachia. Parameters $D_F, D_W$ are diffusion coefficients. All parameters are positive and their interpretation can be found in Campo-Duarte et al. To analyze it, we studied traveling wave solutions of the model, expressing it as a system of ordinary differential equations of dimension four.

We study the stable manifold of the equilibrium points of the fourth-order ordinary differential equations and we find conditions that ensure that the population of Aedes aegypti is totally infected with Wolbachia. In addition, we present a bifurcation analysis including the conditions for the creation and destruction of some heteroclinic orbits that correspond to wavefronts in the equation.

Joint work with:

**Pablo Aguirre**\(^1\), Departamento de Matemática
Universidad Técnica Federico Santa María
Valparaíso, Chile.

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**Star flows: a characterization via Lyapunov functions**, Luciana S. Salgado
lusalgado.sol@gmail.com

In this work, it is presented a characterization of star condition for a $C^1$ vector field based on Lyapunov functions. It is obtained conditions to strong homogeneity for singular sets by using the notion of infinitesimal Lyapunov functions.

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\(^1\)Parcialmente financiado por Proyecto Fondecyt Iniciación 11150306 y Proyecto STIC-AmSud-Conicyt 16STIC-02, e-mail: pablo.aguirre@usm.cl
As an application we obtain some results related to singular hyperbolic sets for flows.

**On the constants of the Bohnenblust-Hille and Hardy-Littlewood inequalities**, Gustavo Araújo and Daniel Pellegrino Departamento de Matemática, Centro de Ciências e Tecnologia, Universidade Estadual da Paraíba, 58.429-500 - Campina Grande, Brazil. and Departamento de Matemática, Universidade Federal da Paraíba, 58.051-900 - João Pessoa, Brazil. gdasara@gmail.com

Results related to summability of multilinear operators date back to the 30’s, when Littlewood proved his seminal $4/3$ inequality. Since then, several different related results and approaches have appeared, as the Hardy–Littlewood inequalities (Q. J. Math., 1934), that can be considered a keystones to the theory of summability of multilinear operators. For $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$, the Hardy–Littlewood inequalities for $m$-linear forms asserts that for $m \geq 2$ and $2m \leq p \leq \infty$ there exists a constant $C_{m,p}^\mathbb{K} \geq 1$ such that, for all continuous $m$-linear forms $T : \ell_p \times \cdots \times \ell_p \to \mathbb{K}$,

$$
\left( \sum_{j_1,\ldots,j_m=1}^{\infty} |T(e_{j_1},\ldots,e_{j_m})|^{\frac{2mp}{mp+p-2m}} \right)^{\frac{mp+p-2m}{2mp}} \leq C_{m,p}^\mathbb{K} \|T\|.
$$

This result was proved by Hardy and Littlewood for bilinear forms and extended to $m$-linear forms by Praciano-Pereira (J. Math. Anal. Appl., 1981). The case $p = \infty$ recovers the seminal Bohnenblust–Hille inequalities (Ann. of Math., 1931). Despite their importance in several fields of mathematics (Quantum Information Theory, Dirichlet series, etc), the optimal constants of the $m$-linear Bohnenblust-Hille and Hardy-Littlewood inequalities are still unknown. In this work, among other results, we show that for $p > 2m(m-1)^2$ the optimal constants satisfying the Hardy–Littlewood inequalities for $m$-linear forms are dominated by the best known constants of the corresponding Bohnenblust–Hille inequalities. For instance, we show that if $p > 2m(m-1)^2$, then

$$
C_{m,p}^\mathbb{C} \leq \prod_{j=2}^{m} \Gamma \left( 2 - \frac{1}{j} \right)^{\frac{j}{2-2j}} < m^{\frac{1-\gamma}{2}},
$$

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Flipped classroom pedagogy in Calculus with English-Medium Instruction in a Confucian Heritage Culture, Natanael Karjanto and Lois Simon natanael@skku.edu

We present flipped classroom pedagogy for a Calculus course. A theoretical framework consists of three components of learning environments: inverted Bloom’s taxonomy for educational learning objectives, an English-medium instruction in Confucian culture for non-native English speakers, and technology adaptation in the instruction. To accommodate various student learning styles and to encourage better learning, we designed three different types of flipped classroom and one instruction type acts as a control. Type A means a fully-flipped class with self-made video recordings and embedding CAS Maxima in the instruction. Type B means a fully-flipped class with readily accessible video recordings made by others from Khan Academy. Type C means only one topic is flipped and type D is the non-flipped, traditional style instruction class. We investigate both quantitative and qualitative aspects of the pedagogy. The result from quantitative analysis indicates that there is no statistically significant difference in terms of the final letter grade means across the four types of instruction as determined by one-way ANOVA ($F(3, 306) = 2.19, p = 0.0892$). On the other hand, a statistically significant difference in the exam score between instruction type A and type C exists ($F(3, 306) = 2.67, p = 0.0477$) and no significant difference can be found among other types of instruction. In both cases, the effect size values ($\eta^2 = 0.0210$ and $\eta^2 = 0.0255$, respectively) suggested small practical significance. Qualitative findings suggest that although students’ engagement and communication with instructors have been improved, there are challenges in improving commitment for out-of-class activities, in minimizing competition but encouraging collaboration among students, and cultivating a sense of curiosity to learn a new technological tool such as CAS Maxima. Furthermore, although no student expressed difficulty in following the instruction in English, it is observed that a large percentage of students is more comfortable providing feedback in their native language.
On Prime Labeling of Graphs, Ebrahim Ghorbani and Sara Kamali
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Let $G$ be a simple graph with $n$ vertices. A prime labeling of $G$ is a labeling of its vertices with distinct integers from $\{1, 2, \ldots, n\}$ in such a way that the labels of any two adjacent vertices are relatively prime. The coprime graph $\mathcal{H}_n$ has vertex set $\{1, 2, \ldots, n\}$ in which two vertices are adjacent if and only if they are relatively prime. So, for an $n$-vertex graph, having prime labeling is equivalent to being a subgraph of $\mathcal{H}_n$. Many properties of $\mathcal{H}_n$ including identifying its subgraphs were studied by Ahlswede and Khachatrian (1994-6), Erdős, Sárközy, and Szemerédi (1980), Szabó and Tóth (1985), Erdős and Sárközy (1969, 1993), and Sárközy (1999).

The notion of prime labeling originated with Entringer. Around 1980 he conjectured that all trees have a prime labeling. Little progress was made on this conjecture until recently that Haxell, Pikhurko, and Taraz (2011) proved that there is an integer $n_0$ such all trees with at least $n_0$ vertices have a prime labeling. Beside that, several classes of graphs have been shown to have a prime labeling, see Gallian’s dynamic survey of graph labeling for more details. One of the graph classes for which the existence of prime labeling is unknown are ladders. Letting $P_n$ denote the path graph on $n$ vertices, the Cartesian product $P_n \times P_2$ is called the $n$-ladder graph. T. Varkey conjectured the following:

**Conjecture.** Ladders have prime labeling.

Partial results on this conjecture have been reported in the literature. Our goal is to prove the conjecture.

On computing the distances to stability for matrices, Nicolas Gillis and Punit Sharma
University of Mons, Belgium punit.sharma$@$umons.ac.be

The stability of a continuous linear time-invariant (LTI) system $\dot{x} = Ax + Bu$, where $A \in \mathbb{R}^{n,n}$, $B \in \mathbb{R}^{n,m}$, solely depends on the eigenvalues of the stable matrix $A$. Such a system is stable if all eigenvalues of $A$ are in the closed left half of the complex plane and all eigenvalues on the imaginary axis are semisimple. It is
important to know that when an unstable LTI system becomes stable, i.e. when it has all eigenvalues in the stability region, or how much it has to be perturb to be on this boundary. For control systems this distance to stability is well-understood. This is the converse problem of the distance to instability, where a stable matrix $A$ is given and one looks for the smallest perturbation that moves an eigenvalue outside the stability region.

In this talk, I will talk about the distance to stability problem for LTI control systems. Motivated by the structure of dissipative-Hamiltonian systems, we define the DH matrix: a matrix $A \in \mathbb{R}^{n,n}$ is said to be a DH matrix if $A = (J - R)Q$ for some matrices $J, R, Q \in \mathbb{R}^{n,n}$ such that $J$ is skew-symmetric, $R$ is symmetric positive semidefinite and $Q$ is symmetric positive definite. We will show that a system is stable if and only if its state matrix is a DH matrix. This results in an equivalent optimization problem with a simple convex feasible set. We propose a new very efficient algorithm to solve this problem using a fast gradient method. We show the effectiveness of this method compared to other approaches such as the block coordinate descent method and to several state-of-the-art algorithms.

**Darboux transformation of periodic potentials**, Andrey Pupasov Maksimov pupasov.maksimov@ufjf.edu.br

We study isospectral families of periodic matrix potentials $\text{Iso}(V)$ using the method of Darboux transformations. Isospectral Darboux transformations define discrete dynamical systems on the manifold $\text{Iso}(V)$. Darboux transformations between diagonal and non-trivially coupled potentials can be constructed using second order transformations with complex factorization energies. In the case of matrix potentials it is convenient to substitute a system of $N$ second order Schrodinger equations by a system of $2N$ first order equations. In terms of symplectic geomety:

1) Matrix seed solution $U(x)$ uniquely defines a family of $N$-dimensional linear subspaces $S(x)$ (seed subspaces) of $2N$-dimensional symplectic complex space. Evolution of $S(x)$ is defined by a linear Hamiltonian system.
2) Darboux transformation results in a symmetric transformed potential if and only if the seed subspace is a Lagrangian plane.
3) Transformed potentials are non-singular if $S(x)$ is transversal to the vertical
Lagrangian plane.

Our main result is an explicit construction of the isospectral Darboux transformation between diagonal and coupled periodic potentials. Transformed potentials are periodic if and only if the seed subspace is an invariant subspace of some power of the monodromy operator at the factorization energy $\mathcal{E}$, $M^n(\mathcal{E})S(0) \subset S(0)$. The transformed potential is isospectral to the initial one and its period is equal to $nT$. Diagonal potential can be transformed to a coupled one if there is at least one eigenvalue of $M^n(\mathcal{E})$, $\mathcal{E} \in \mathbb{C}$ with a multiplicity greater than 1. Given a pair of scalar periodic potentials $V_1(x)$, $V_2(x)$ with Lyapunov exponents $\beta_1(E)$, $\beta_2(E)$, solutions of equation $\beta_1(E) - \beta_2(E) = 0$ are related with topology of $\text{Iso}(\text{diag}(V_1, V_2))$. If there are no complex solutions, all matrix potentials which are isospectral with $\text{diag}(V_1, V_2)$ should be diagonal, thus $\text{Iso}(\text{diag}(V_1, V_2)) = \text{Iso}(V_1) \times \text{Iso}(V_2)$. A possibility to control coupling between scattering channels in the case of periodic potentials can be applied to design 1D structures in quantum optics. We give explicit examples of coupled-channel partners for the matrix Kroning-Penney and finite zone potentials. Finally, we discuss relations of the obtained results and the Moser-Trubowitz isomorphism in the case of multiparticle Neumann oscillators.

**Small quotients of finite linear groups**, Thomas Michael Keller *Texas State University*

Let $G$ be a finite group and $V$ a finite faithful irreducible $G$-module. In joint work with Yong Yang we examined upper bounds for the size of the abelian quotient $|G/G'|$ in terms of the action on $|V|$. It had been known that $|G/G'|$ is bounded above by $|V|$, and we strengthened this by proving that $|G/G'|$ is bounded above by the largest orbit size of $G$ on $|V|$. We also studied what can be said when equality holds. Taking the next step we then considered what might be said for the class 2 quotient of the group, i.e., the largest factor group of $G$ which is of nilpotence class 2. Do similar results hold for this larger quotient?

**Fighting against resistant bacteria, how long will antibiotics help?**, Damián A. Knopoff
A mathematical model for bacterial growth, mutations, horizontal transfer and development of antibiotic resistance is presented. The model is based on the so-called kinetic theory for active particles that is able to capture the main complexity features of the system. Bacterial and immune cells are viewed as active particles whose microscopic state is described by a scalar variable. Particles interact among them and the temporal evolution of the system is described by a generalized distribution function over the microscopic state. The model is derived and tested in a couple of case studies in order to confirm its ability to describe one the most fundamental problems of modern medicine, namely bacterial resistance to antibiotics.

**Ordinary differential equations in Brazil: Genesis and Recent Developments,** Jorge Sotomayor  
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A. An outline of the transition from the work of Henri Poincaré (France, around 1881) to that of Mauricio Peixoto (Brazil, around 1962), touching on the contributions of Alexander Andronov, Lev Pontrjagin and Evgenia Leontovich (Russia, years 1935 - 1955) with a brief interlude in USA with Solomon Lefschetz (around 1950).


A3. Lefschetz books and the divulgation in the West of the work around A2.


B1. A glimpse into the works accomplished at Peixoto’s Seminar.

C. On the repercussion of Peixoto’s work in Brazil and abroad.

C1. 1981, Structural Stability and Bifurcations of bidimensionais EDOs. The
work of Carlos Gutierrez.

\( C_2 \). 1991, Principal curvature configurations on surfaces in euclidean space.

\( C_3 \). 2009, Extension in one dimension and co-dimension on the domain and co-domain in \( C_2 \).

\( D \). On the present and future of the research on the Qualitative Theory of Ordinary Differential Equations and Dynamical Systems in Brazil. Artur Óvila Fields Medal achievement.

Boundary-value problems for the third order loaded equation with characteristic type-change lines, Umida Baltaeva umida_baltayeva@mail.ru

The theory of mixed type equations is one of the principal parts of the general theory of partial differential equations. Moreover, in recent years it has become increasingly important to investigate a new class of equations, known as loaded equations, as a direct result of issues with the optimal control of the agro economical system, long-term forecasting and regulating the level of ground water and soil moisture.

On the other hand, fractional derivatives (e.g., Riemann-Liouville and Caputo) were widely used to model the complex phenomenon in science and engineering practice. In this connection, linear fractional partial differential equations models are commonly encountered in applied mathematics and engineering. However, we would like to note that boundary-value problems for third-order loaded equations of a mixed types with the integro-differential operator are not well studied. Hence, the main aim of the work is to establish unique solvability certain generalized boundary-value problem for the linear loaded, integro-differential equation of the third order, with the mixed operators.

Spectral Decomposition and \( \Omega \)-Stability of Flows with Expanding Measures, Keonhee Lee Chungnam National University, Korea khlee@cnu.ac.kr

A flow \( \phi \) on a compact metric space \( X \) is said to be measure expansive if for any \( \phi \)-orbit vanishing Borel probability measure \( \mu \) on \( X \), there is \( \delta > 0 \) such that
\( \mu(\Gamma^\phi_\delta(x)) = 0 \) for all \( x \in X \). In this talk, we introduce a concept of measure expanding which is slightly different with that of measure expansiveness. We say that \( \phi \) is measure expanding if for any Borel probability measure \( \mu \) on \( X \), there is \( \delta > 0 \) such that \( \mu(\Gamma^\phi_\delta(x) \setminus \phi_R(x)) = 0 \) for all \( x \in X \). We prove that any invariantly measure expanding flow with shadowing has the spectral decomposition, and give an example to show that a measure expansive flow with shadowing does not have the spectral decomposition. Moreover, we claim that an integrated flow of a \( C^1 \) vector field without singularities on a compact \( C^\infty \) manifold is \( C^1 \) stably invariantly measure expanding if and only if it satisfies Axiom A and no-cycle condition. This is joint work with N.T. Nguyen.

Effects of baffle on reducing liquid sloshing in a tank, Neelam Choudhary and Swaroop Nandan Bora Bennett University, India and IIT Guwahati, India neelam.choudhary$@$bennett.edu.in and 87neelu$@$gmail.com

For a specially designed circular cylindrical container filled with an inviscid, incompressible and homogenous liquid, if an annular baffle is attached to the outer cylinder wall in the annular region of the cylinder at some depth, the natural frequencies and the response of the liquid in the container undergo a drastic change. Such an introduction of a baffle divides the liquid region into four. Boundary value problems are set up for the potential in each of these regions, and with the help of the matching conditions across the virtual interfaces, we set up a system of linear equations by solving which we determine the natural frequencies. A partly covered interface shifts the natural frequency above and away from the control frequency of the vehicle, in which the liquid-filled container is placed, which results in the reduction of sloshing mass participating in the dynamic motion of the system. The fundamental natural frequency of the liquid is determined for different width and positions of the baffle in addition to different configurations. All our observations are supported by relevant graphs.

Mixed perverse sheaves on flag varieties of Coxeter groups, Pramod N. Achar,
Simon Riche and Cristian Vay Louisiana State University, U.S.A., Université Clermont Auvergne, France and Universidad Nacional de Córdoba, Argentina
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This communication is based on the homonymous work [arXiv:1802.07651] where we construct an abelian category of “mixed perverse sheaves” attached to any realization of a Coxeter group, in terms of the associated Elias–Williamson diagrammatic category. This construction extends previous work of the first two authors, where we worked with parity complexes instead of diagrams, and we extend most of the properties known in this case to the general setting. As an application we prove that the split Grothendieck group of the Elias–Williamson diagrammatic category is isomorphic to the corresponding Hecke algebra, for any choice of realization.

The delta-subharmonic functions of completely regular growth in the upper half-plane, Iryna Kozlova and Kostiantyn Maliutin angelinarena@yahoo.com

The spaces of delta-subharmonic functions of completely regular growth in the upper half-plane with respect to the function of growth is considered. The theory of the analytical functions of completely regular growth (c.r.g.) in the half-plane are generalized in two directions: 1) the growth of functions is measured with respect to an arbitrary growth function $\gamma(r)$, satisfying the condition $\gamma(2r) \leq M \gamma(r)$, for some $M > 0$ and all $r > 0$; 2) the spaces of delta-subharmonic functions of c.r.g. in the half-plane are introduced. Denote by $J\delta$ the space of proper delta-subharmonic functions in the upper half-plane.

The function $v \in J\delta$ is called a function of finite $\gamma$-type if there exist constants positive $A$ and $B$ such that for all $r > r_0$

$$T(r, v) \leq A \frac{r}{r} \gamma(Br).$$

We denote the corresponding class of $\delta$-subharmonic functions of finite $\gamma$-type by $J\delta(\gamma(r))$.

A positive measure $\lambda$ has finite $\gamma$-density if there exist positive constants $A$
and $B$ such that for all $r > r_0$

$$N(r, \lambda) := \int_{r_0}^{r} \frac{\lambda(t)}{t^3} dt \leq \frac{A}{r} \gamma(Br).$$

The Fourier coefficients of a function $v \in J_{\delta}$ are defined as

$$c_k(r, v) = \frac{2}{\pi} \int_{0}^{\pi} v(re^{i\theta}) \sin k\theta \, d\theta, \quad k \in \mathbb{N}.$$

**Definition.** A function $v \in J_{\delta}$ is said to be a function of a completely regular growth with respect to $\gamma(r)$ if there is the limit

$$\lim_{r \to \infty} \frac{1}{\gamma(r)} \int_{\eta}^{\varphi} v(re^{i\theta}) \sin \theta \, d\theta.$$

for all $\eta$ and $\varphi$ from the interval $[0, \pi]$.

Let us denote the corresponding class of $\delta$-subharmonic functions of a c.r.g. with respect to $\gamma(r)$ by $J_{\delta}(\gamma(r))^o$. Let $\widetilde{L}^{\infty}[0, \pi]$ be the Banach subspace of $L^{\infty}[0, \pi]$ generating by the family of characteristic functions of all intervals from $[0, \pi]$. The following result is valid.

**Theorem.** Let $v \in J_{\delta}$. The following properties are equivalent: (i) $v \in J_{\delta}(\gamma(r))^o$; (ii) $v \in J_{\delta}(\gamma(r))$ and for all $k \in \mathbb{N}$ there exists $\lim_{r \to \infty} \frac{c_k(r, v)}{\gamma(r)} = c_k$; (iii) the measure $\lambda_-(v)$ has a finite $\gamma$-density and for any function $\psi$ from $L[0, \pi]$ there exists a limit

$$\lim_{r \to \infty} \frac{1}{\gamma(r)} \int_{0}^{\pi} \psi(\theta) v(re^{i\theta}) \sin \theta \, d\theta.$$

Existence and symmetry of solutions to overdetermined boundary problems on Riemannian manifolds, Miguel Domínguez-Vázquez, Alberto Enciso and Daniel Peralta-Salas *ICMAT-CSIC, Madrid*

In the theory of partial differential equations, an overdetermined boundary value problem is one for which both Dirichlet and Neumann conditions are imposed simultaneously on the boundary of a domain. In a seminal paper in 1971,
J. Serrin proved that the only bounded domains of the Euclidean space that admit solutions to certain overdetermined elliptic problems are balls. Since then, multiple extensions of this result have been obtained, some of them showing intriguing connections with the theories of constant mean curvature and isoparametric hypersurfaces.

One of the outstanding questions in the area is whether, in a general Riemannian manifold, there are domains that admit solutions to overdetermined problems for a large class of nonlinear elliptic equations. Another open problem is to decide whether the analog of Serrin’s result for the rank one symmetric spaces holds or not.

In this talk I will report on a recent joint work with A. Enciso and D. Peralta-Salas where we provide a positive answer to the first question. I will also explain how our techniques allow us to make progress on the second-mentioned problem, by obtaining certain symmetry results in harmonic spaces.

**Is the ”delta” a difference? On the prehistory of delta-epsilon notation**, João F. Cortese joaocortese@gmail.com

It is generally acknowledged that delta-epsilon proofs methods were introduced in the 19th century by Cauchy and Weierstrass – the first introducing its notation and the latter developing the delta-epsilon method for the definition of a limit (cf. for example Sinkevich 2016, Grabiner 1983 and Cajori 1993).

In this communication I would like to show how a relational aspect between two variables similar to the one between the modern delta-epsilon methods appear in the work of Blaise Pascal, namely in his Lettres de A. Dettonville. Pascal’s method allows a comparison between two kinds of “indivisibles”: the small portions and the differences between the sums of small portions and the corresponding magnitudes. Even if Breger (2008) claims that “the connection between infinitesimals and what we now call epsilontics was obvious enough for 17th-century mathematicians”, a precise analysis of Pascal’s contribution and is specificity is necessary (see also Whiteside 1961).

Furthermore, I will show that in Pascal’s recent discovered manuscripts (Descotes 2010), he uses the letter $d$ for the division of the base and the letter $e$ for the division of the curve, in a writing that resembles the characters of $\delta$ and $\epsilon$. 120
Something similar will be found in Leibniz’s *Quadratura Arithmetica*.

This communication will then present a hypothesis that early works on dependent variables by Pascal, close to $\delta-\epsilon$ methods, could be related to an idiosyncratic notation of divisions that reminds the writing of $\delta$ and $\epsilon$.

**Prices under differentiation in networks**, João P. Almeida and Alberto A. Pinto
jpa@ipb.pt

We develop a theoretical framework to study the location-price competition in a Hotelling-type network game, extending the Hotelling model with linear transportation costs from the line (city) to a network (town). We show the existence of a pure Nash equilibrium price if, and only if, some explicit conditions on the production costs and on the network structure hold. Furthermore, we prove that the local optimal localization of the firms are at the cross-roads of the town.

**The representation of consumer’s preferences by marginal rates of substitution - An alternative approach represented by PDEs**, Jorge M. Marques
jmarques@fe.uc.pt

In this talk we are interested in the representation of consumer’s preferences from the differentiable point of view. In this setting we consider marginal rates of substitution to describe smooth preferences on the multidimensional space of goods. The main goal is to provide an analytic method to construct the indifference map. Then we solve a first order system of partial differential equations (PDEs) given by a vector field that holds the integrability conditions. These conditions impose symmetry properties in the underlying preferences. We obtain results relating to linear, quasi-linear, separable, homothetic, Cobb-Douglas, and CES preferences. Our alternative approach to understand consumer behavior is connected with the formulation concerning the representability of preferences by smooth utility functions.

**Logarithmic differential forms on singular varieties**, A.G. Aleksandrov Russian Academy of Sciences, Russia ag_aleksandrov@mail.ru
The concept of logarithmic differential forms with poles along a reduced (i.e., having no multiple components) divisor given on a complex smooth manifold appeared in the early 1960s in relation to studies of Hodge structure and the Gauss-Manin connection in the cohomology of algebraic varieties. More precisely, a meromorphic form $\omega$ on a complex manifold $M$ with poles along a divisor $D \subset M$ is traditionally called logarithmic if both forms $\omega$ and $d\omega$ have poles only along the divisor $D$ and of order not greater than 1. The corresponding sheaves are usually denoted by $\Omega^p_M(\log D)$, $p \geq 0$. For instance, P. Deligne, N. Katz, and Ph. Griffiths considered this notion for a union of smooth subvarieties with normal crossings, K. Saito studied the case of arbitrary reduced divisors, etc.

We develop an alternative approach to the study of properties of logarithmic differential forms; it is mainly based on an original interpretation of the notion of a logarithmic differential form in terms of the classical de Rham lemma adapted to the study of differential forms given on singular hypersurfaces with arbitrary singularities. In the talk we show how considerations of this kind enable us to extend the notion of a logarithmic differential form to the case of effective Cartier divisors, or reduced complete intersections, or, more generally, to the case of Cohen-Macaulay subvarieties given on varieties with singularities. As examples, we also compute generators of the modules of logarithmic differential forms and some useful invariants in the case of divisors given on complete intersections, on determinantal varieties, normal and non-normal spaces, and others.

In a modern form the concept of logarithmic differential forms with poles along a divisor $D$ in a complex manifold $M$ has appeared in 1960s in relation with studies of Hodge structure and Gauss-Manin connection for singular varieties. More precisely, a meromorphic differential form $\omega$ with poles along $D$ is called logarithmic if $\omega$ and $d\omega$ have at worst simple poles only along the divisor $D$. The corresponding sheaves are usually denoted by $\Omega^p_M(\log D)$, $p \geq 0$. For instance, P. Deligne, N. Katz and Ph. Griffiths considered this notion for a union of smooth subvarieties with normal crossings, K. Saito and his successors studied the case of arbitrary reduced divisors, etc.

In his previous works, the author elaborated an alternative approach in studies of this subject, which is based on an original interpretation of the notion of logarithmic differential forms in terms of a modified version of the classical de Rham lemma adapted to the case of singular varieties. In particular, this approach
can be used to create the theory of logarithmic differential forms for Cartier divisors in singular varieties. As examples, we also compute generators of the modules of logarithmic differential forms and some useful invariants in the case of divisors given on complete intersections, on determinantal varieties, normal and non-normal spaces, and others.

The purpose of the talk is to give an elementary introduction to the theory of residue of logarithmic and multi-logarithmic differential forms, and to describe some of the less known applications of this theory, developed by the author in the past few years. In particular, we briefly discuss the notion of residue due to H. Poincaré, J. de Rham, J. Leray and K. Saito, and then obtain an explicit description of the modules of regular meromorphic differential forms in terms of residues of meromorphic differential forms logarithmic along hypersurface or complete intersections with arbitrary singularities. We also construct a complex \( \Omega_S^\bullet (\log C) \) of sheaves of multi-logarithmic differential forms on a complex analytic manifold \( S \) with respect to a reduced complete intersection \( C \subset S \), and define the residue map as a natural morphism from this complex onto the Barlet complex \( \omega_C^\bullet \) of regular meromorphic differential forms on \( C \). This implies that sections of the Barlet complex can be regarded as a generalization of the residue differential forms defined by Leray. Summarizing all of these ideas we describe an intrigued generalization of the notion of multi-logarithmic differential forms to the case of Cohen-Macaulay varieties.

Diagrammatic reducibility of presentations and equations over groups,
Jonathan A. Barmak and E. G. Minian  
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If \( g_1 \) and \( g_2 \) are two elements of different order in a group \( G \), then the equation \( g_1xg_2x^{-1} = 1 \) has no solutions in \( G \). Moreover, it has no solutions in any overgroup \( H \supseteq G \) of \( G \). In all the examples known of equations \( g_1x^{\epsilon_1}g_2x^{\epsilon_2}\ldots g_nx^{\epsilon_n} = 1 \) with no solutions in any overgroup, the total exponent \( \sum \epsilon_i \) of \( x \) is 0. In this talk we will study equations in two variables \( x, y \) in which the total exponent of \( x \) and of \( y \) is 0. We will use a new test which implies asphericity and diagrammatic reducibility of presentations.
Assessment of the effect of temperature on the basic offspring number of Sand fly population, the vector of leishmaniasis, Schehradad Selmane scselmane@yahoo.fr

The Phlebotomine sand flies, small biting flies found in tropical, subtropical and temperate regions throughout the world, are incriminated in the transmission of leishmaniases, a set of parasitic diseases of zoonotic origin. The parasites are transmitted to human through the bite of infected female Phlebotomine sand flies. The females acquire Leishmania parasites and infection starts when they fed on an infected mammalian host on blood meal intake which allows the females to obtain the protein necessary to develop eggs. Therefore, monitoring in vectors represents one of the important tools to control the spread of the parasites. The aim of this work was to estimate the size of phlebotomine sand flies population and to gauge the impact of temperature on the size of this population and finally to address the question on how the temperature affects leishmaniasis incidence. To this end, we developed a structured mathematical model based on sand fly life cycle which encompasses four successive stages, namely, egg, larva, pupa, and adult. The global stability of the sand fly free equilibrium and the endemic equilibrium are proved and the basic offspring number, which represents the capacity of sand fly reproduction, is derived. To evaluate the influence of temperature on the developmental rates of the sand fly, we considered the entomological parameters involved in the basic offspring number, as temperature dependent. This allowed us to assess the effect of temperature on the size of the sand fly population. The sand fly population cannot be maintained in an area when the temperature is bellow $15^\circ C$ or when it exceeds $32^\circ C$, and the optimum temperature for reaching the high sand flies population densities was found to be $28^\circ C$. This knowledge would assist decision-makers in identifying and selecting the most suitable strategies from the available mechanisms timely.

Holonomy of the Bismut connection on Vaisman solvmanifolds, Adrián Andrade and Raquel Villacampaandrada@famaf.unc.edu.ar

A Hermitian manifold $(M, J, g)$ is called locally conformal Kähler (LCK) if each point of $M$ has a neighborhood where the metric $g$ is conformal to a Kähler
metric with respect to $J$. Equivalently, there exists a closed 1-form $\theta$ on $M$ such that $d\omega = \theta \wedge \omega$, where $\omega$ denotes the fundamental 2-form associated to $(J, g)$, defined by $\omega(\cdot, \cdot) = g(J \cdot, \cdot)$. The 1-form $\theta$ is called the Lee form.

If the Lee form is parallel with respect to the Levi-Civita connection associated to $g$, the LCK structure is called Vaisman. The family of Vaisman manifolds is very important and they have been deeply studied since the seminal work of I. Vaisman in the '80s.

On the other hand, any Hermitian manifold $(M^{2n}, J, g)$ admits a unique connection $\nabla^b$ which satisfies $\nabla^b J = 0$, $\nabla^b g = 0$ and its torsion $T^b$ es totally skew-symmetric, that is, $c(X, Y, Z) = g(X, T^b(Y, Z))$ is a 3-form on $M$. The connection $\nabla^b$ is called the Bismut connection and it has holonomy contained in $U(n)$.

In this talk we study the holonomy of the Bismut connection on Vaisman solv-manifolds, that is, on compact quotients $\Gamma \backslash G$, where $G$ is a simply connected solvable Lie group and $\Gamma$ is a discrete subgroup of $G$. We assume that the Vaisman structure on $\Gamma \backslash G$ is induced by a left-invariant Vaisman structure on $G$.

In this case, we prove that the (restricted) holonomy of $\nabla^b$ reduces to a one-dimensional subgroup of $U(n)$, which is not contained in $SU(n)$. To show this, we exhibit first some general results about the Bismut connection on arbitrary Vaisman manifolds, and then we use the characterization of unimodular solvable Lie algebras that admit Vaisman structures.

This is a joint work with Raquel Villacampa (Zaragoza, España).

**Boundary feedback stabilization of shock steady-states for hyperbolic partial differential equations: Burgers and Saint-Venant equations.** Amaury Hayat, Georges Bastin, Jean-Michel Coron and Peipei Shang hayat@ljll.math.upmc.fr

Shocks are a well-known phenomenon that arises naturally in nonlinear hyperbolic PDEs, contributing to the richness of this class of equations. In fluids, they correspond to physical phenomena, one of the most famous being the hydraulic jump, often occurring in water channel or in the atmosphere. However, when stabilizing the steady-states of such equations, most studies, if not all, focus only on classical steady-states, ignoring shock steady-states. Stabilizing shock steady-states has both a large theoretical interest and a practical interest as these shocks
are sometimes engineered on purpose (e.g. hydraulic jumps to protect hydraulic installations).

We introduce a method to stabilize a steady-state with a shock for the inviscid Burgers equation (resp. the Saint-Venant equations) and we prove that with simple pointwise feedbacks we can achieve exponential stability of both the state and the shock location with an arbitrary decay rate. This method relies on two steps: first an adapted change of variable to transform a scalar equation (resp. 2 × 2 system) with a moving shock into a 2 × 2 (resp. 4 × 4) strictly hyperbolic system on a fixed domain coupled with an ODE. Then, the construction of a particular Lyapunov function that uses the state of the system and the coupling between the variable of the ODE and the state of the system. This method is applied to homogeneous equations, but could be generalized for inhomogeneous equations.

**Global properties of systems on compact manifolds**, Giuliano A. Zugliani and Jorge G. Hounie giuzu2000@gmail.com

We consider the linear operator \( \mathbb{L} = d_t + c(t) \partial_x \), where \( x \in S^1 \) and \( c \) is a closed smooth 1-form on a compact manifold \( M \), that may be regarded as the first operator of the complex induced by a locally integrable structure of tube type and co-rank one.

Denote by \( a \) and \( b \), respectively, the real and imaginary parts of \( c \). We first discuss the problems of the global solvability and the global hypoellipticity of \( \mathbb{L} \) when \( b \) is a real analytic form and \( M \) has a real analytic structure. They are both characterized by geometric properties and diophantine conditions related to the behavior of \( a \) on the critical set of \( b \).

For a smooth form \( b \), the characterizations also hold if the structure is of Mizohata type. We also present results for smooth forms defined on the torus or having rank equal to 1.

We thank FAPESP for the financial support.

**A regularization method based on an augmented Lagrangian approach for parameter identification**, Juan P. Agnelli, Adriano De Cesaro and Antonio Leitão agnelli@famaf.unc.edu.ar

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We propose and analyze a solution method for parameter identification problems modeled by ill-posed nonlinear operator equations in Banach spaces, where the parameter function to be identified is known to be a piecewise constant function.

A level-set approach is used to represent the unknown parameter, and a corresponding Tikhonov functional is defined (on the space of level-set functions). Additionally, a suitable constraint is enforced, resulting that our Tikhonov functional is to be minimized over a set of piecewise constant level-set functions. Summarizing, the original parameter identification problem is rewritten as in the form of a constrained optimization problem, which is solved here by means of an augmented Lagrangian type method.

We prove existence of zero duality gaps and existence of generalized Lagrangian multipliers for our Lagrangian approach. Moreover, we prove convergence and stability of our parameter identification method, i.e. our solution method is a regularization method.

Additionally, a primal-dual algorithm is proposed to compute approximate solutions of the original inverse problem, and its convergence is provided. Numerical examples applied to a 2D diffuse optical tomography benchmark problem demonstrate the viability of the proposed approach.

**Some on the Annihilator Ideal Graph**, Mohammad Javad Nikmehr K. N. Toosi University of Technology, Iran nikmehr@kntu.ac.ir

The annihilator ideal graph of $R$, denoted by $\Gamma_{\text{Ann}}(R)$, is a graph whose vertices are all non-trivial ideals of $R$ and two distinct vertices $I$ and $J$ are adjacent if and only if $I \cap \text{Ann}(J) \neq (0)$ or $J \cap \text{Ann}(I) \neq (0)$. In this paper, we show that if $R$ is reduced, then $\Gamma_{\text{Ann}}(R)$ is complete multipartite. Also, some results on the annihilator ideal graphs with finite clique numbers are given. Moreover, some properties such as connectivity, diameter, girth and etc. of a subgraph induced by ideals with non-zero annihilators are studied.

**Prediction law of fractional Brownian motion**, Tommi Sottinen and Lauri Viitasaari Department of Mathematics and Statistics, University of Vaasa, P.O. Box 127
We calculate the regular conditional future law of the fractional Brownian motion with index $H \in (0, 1)$ conditioned on its past. We show that the conditional law is continuous with respect to the conditioning path. We investigate the path properties of the conditional process and the asymptotic behavior of the conditional covariance.


**Interfaces for 2d long-range Ising Models**, Loren Coquille and Arnaud Le Ny
Université Grenoble Alpes, France, Université Paris-Est, France, and TU/e Eindhoven, Netherlands loren.coquille@gmail.com and arnaud.le-ny@u-pec.fr

In this short presentation/poster, we propose to describe recent progress concerning interface states for 2d long range Ising models. These are models of ±1-Ising spins with polynomially decaying pair potentials whose decay rate is sometimes interpreted as a continuous dimension parameter that allows to tune continuously energy estimates from some dimension $d$ to dimension $d + 1$ in the standard nearest neighbour (*n.n.*) case. One dimensional such models with slow interaction decay are e.g. know to exhibit phase transition and Gaussian fluctuations of the so called interface state, got by weak limit of a mixed ± boundary condition. The limiting states is there, as in 2d for *n.n.* models, not extremal nor translation-invariant. It is known that in 3d such limiting states can be shown to be extremal and non-translation invariant, manifesting there some rigidity of the interface and called Dobrushin states. A natural question for long-range models is to investigate whether very slow decay could also tune the dimension to get rigidity in such 2d models and whether it has an impact on the Gaussian fluctuations.
In order to describe our results, we propose the following scheme:

- To review the full convex picture got for the $2d$-n.n. ferromagnetic case.
- To review essential results concerning phase transition for long range models on $\mathbb{Z}^d$.
- To show that for long range $2d$-Ising models also there can not exist Dobrushin states.
- To discuss various descriptions about the fluctuations of the interface (AH theorem, surface tension, step free energy, Lévy bridges etc.) for these long-range models.

Effect of Probability Sampling Techniques on Gaussian Naive Bayes Classification for Neonatal Survival, Ezekiel Gyimah and Atinuke Adebani Kwame Nkrumah University of Science and Technology, Kumasi, Ghana

In data sparse countries, statistics are not sufficiently complete to allow reliable classification of neonatal survival, therefore appropriate sampling methods on existing data are required for proper prediction of neonatal mortality and survival. These methods should result in unbiased, accurate and precise estimates of population parameters. This study focused on the effect of probability sampling techniques – simple random sampling, stratified sampling and systematic sampling – on the performance of Gaussian Naive Bayes classification algorithm for predicting neonatal survival. Performance evaluation for the different sampling techniques applied was done using the 5-fold cross validation, leave-one-out cross validation and balanced error rate procedures. The study considered 520 neonatal data (260 mortality and 260 survival) sampled from 3869 discharges (survivals) and 858 deaths available from Maternity and Baby Unit (MBU) at a Teaching Hospital in Ghana, using the three sampling techniques. After pre-processing the data, three out of an initial seven predictors were used in the analysis – age of baby, maternal age and parity. Results obtained showed the systematic sampling technique as the best performer for predicting neonatal survival across all performance evaluation procedures.
Cauchy extension of Hilbert C*-modules, Kourosh Nourouzi and Ali Reza nourouzi@kntu.ac.ir

In this talk we introduce the Cauchy extension of Hilbert C*-modules. We show that the Cauchy extension preserves Morita equivalence of Hilbert C*-modules. It is also shown that the Cauchy extension of Hilbert C*-modules preserves adjointable operators and partial isometries.

On the bondage numbers of conjugacy class graphs, Zohreh Mostaghim Iran University of Science and Technology, Iran mostaghim@iust.ac.ir

Let $G$ be a finite group. Let $\Gamma(G)$ be the conjugacy class graph of $G$, that its vertex set is the set of non-central conjugacy classes of $G$, and two distinct vertices $C$ and $D$ are connected by an edge if and only if $|C|$ and $|D|$ have a non-trivial common divisor. The bondage number of a graph is the minimum number of edges whose removal results in a graph with larger domination number. In this paper, we study the bondage number for groups $G$ whose conjugacy class graphs $\Gamma(G)$ do not contain a subgraph $K_4$ or $K_5$. Also, we compute some invariants related to conjugacy class graphs for some classes of finite groups.

Erdosian functions and associated L-functions, Suraj S. Khurana and Tapas Chatterjee suraj.khurana@iitrpr.ac.in

A function $f$ is said to be Erdősian modulo $q$ if it is a periodic arithmetic function with period $q$ which takes values in the set \{-1, +1\} everywhere except at $q$ where it is 0. A folklore conjecture of Erdős asserts that for such a function $f$, $L(1, f)$ never vanishes where $L(s, f)$ is the L-function attached to $f$. In this presentation we shall see that if the average order of Dirichlet inverse is not “too big” then the Erdős conjecture is true. Further we discuss an orthogonality like relation satisfied by these functions and recover the classical Gauss formula for the digamma function at rational arguments. At the end we use analytic tools to prove that the Dirichlet L-series associated with completely multiplicative Erdősian functions only takes positive values at positive integers.
A Tale of Two Cones since Hilbert’s 1888 theorem, Charu Goel charugoelin@yahoo.com

The relationship between the cone of positive semidefinite (psd) real forms and its subcone of sum of squares (sos) of forms is of fundamental importance in real algebraic geometry and optimization, and has been studied extensively. The study of this relationship goes back to the 1888 seminal paper of Hilbert, where he gave a complete characterisation of the pairs \((n, 2d)\) for which a psd \(n\)-ary \(2d\)-ic form can be written as sos. In this short communication, we will show how this relationship changes under the additional assumptions of symmetry on the given forms.

We will present our recent results giving the analogues of Hilbert’s characterisation for symmetric and even symmetric forms respectively. Along the way, I will also discuss briefly how test sets for positivity of symmetric polynomials play an important role in establishing these analogues.

(Joint work with S. Kuhlmann and B. Reznick)

Approximated solution of modified Reve’s puzzle, Jinwoo Seo and Jongmin Kim skjin0702@gifted.or.kr

The N-pegged Tower of Hanoi is a popular puzzle which asks for the minimum number of moves required to move disks from one peg to another preserving the order of their size. In particular, 4-pegged Tower of Hanoi, called Reve’s puzzle also has been studied by numbers of research groups due to its simple form and important properties. In 1941, Frame and Stewart suggested algorithm which predicts the general solution of multi-peg Tower of Hanoi, but the optimality of that algorithm remains as a conjecture. In this research, we studied the approximate solution of modified Reve’s puzzle which had never studied before in best of our knowledge. The puzzle has an additional rule; four pegs are placed in a square form and movements of disks between pegs distributed diagonally are disallowed. We suggest a system of recurrence formulas based on Frame-Stewart algorithm, which predicts the general term of the minimum number of moves required. However, because of complexity of the recurrence formulas, we provide an approximation of the general term through numerical methods. Furthermore,
we draw out intriguing results which are completely different to those of original puzzle.

**On extension of smooth local maps of Banach spaces and applications**, Victoria Rayskin and Genrich Belitskii  

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A known classical method of extension of smooth local maps of Banach spaces uses smooth bump functions. However, such functions are absent in the majority of infinite-dimensional Banach spaces. This is an obstacle in the development of local analysis, particularly in the questions of extension of local maps onto the whole space. We will present a new approach that substitutes bump functions with special maps, which we call blid maps. In addition to smooth spaces, blid maps also allow to extend smooth local maps from non-smooth spaces, such as $C^q[0,1], q = 0, 1, \ldots$. We will show how to reconstruct a map from its derivatives at a point, for spaces possessing blid maps. We will also show how blid maps can assist in finding global solutions to cohomological equations having linear transformation of argument. In particular, we will discuss application of blid maps to conjugation/linearization of maps on Banach spaces, preserving smoothness in both: variables and parameters.

**Invariants of fundamental representations of SL$n$ and colored hypergraphs**,  
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Classical invariant theory studies invariants of linear groups and their relations. The action of SL$_2$ on symmetric tensors was a main subject in the nineteenth century. That is to say, a low dimensional acting group was combined with a relatively complicated representation of the group. We pursue the opposite strategy by observing the action of higher dimensional special linear groups on antisymmetric tensors, i.e. we consider invariants of (sums of) fundamental representations of SL$_n$. To any such invariant, we associate a colored hypergraph and we allow certain operations on these hypergraphs. This allows us to substantially simplify the
hypergraphs and to use graph theoretic results for a very effective determination of the ring of invariants. This directly generalizes the fundamental theorems for systems of vectors and linear forms. As an explicit example, we observe the group $SL_4$.

A Neighborhood Hypothesis Test for Functional Data with an Application in Investigating Agreement in Drug Sensitivity Profiles, Leif Ellingson, Dhana-malee Bandara and Souparno Ghosh leif.ellingson@ttu.edu

A common problem arising when analyzing high-dimensional or functional data is that estimates of the covariance are not of full rank, resulting in the inverse being degenerate. Munk et al. (2008) applied the idea of a neighborhood hypothesis test to the one- and multi-sample problems for functional data by deriving a test statistic to determine whether a group of means are approximately equal. More precisely, they tested whether the means were within a predetermined distance to each other. Unfortunately, in many applications, this pre-determined distance is difficult to both specify and interpret.

In this presentation, we present a modified test for determining whether the distance between a mean and a hypothesized function is less than a proportion of the total population variance. We will present an asymptotically normal test statistic that does not require the sample covariance matrix to be invertible. In simulation studies of the power of the procedure, we will show that this procedure has good statistical performance even with relatively small sample sizes despite it originating from asymptotic theory. We will conclude with an application comparing dose-response profiles from the publicly available Genomics of Drug Sensitivity in Cancer (GDSC) and Cancer Cell Line Encyclopedia (CCLE) databases to assess the level of agreement between them.

Regularity estimates for fully nonlinear parabolic models, João Vitor da Silva and Eduardo V. Teixeira jdasilva@dm.uba.ar

In this Lecture, we discuss sharp regularity estimates for viscosity solutions of
parabolic equations as follows

\[
\frac{\partial u}{\partial t} - F(x, t, Du, D^2u) = f(x, t) \quad \text{in} \quad Q_1 = B_1 \times (-1, 0],
\]

where \( F \) is a second order fully nonlinear (uniformly elliptic) operator with merely measurable coefficients (under small enough oscillation in an appropriated sense) and \( f \in L^{p,q}(Q_1) \), i.e., an anisotropic Lebesgue space with exponents \( p, q \in [1, \infty) \) fulfilling \( \kappa(n, p, q) = \frac{n}{p} + \frac{2}{q} \in (0, 1) \). Under such assumptions, we establish local \( C^{1+\alpha, \frac{1+\alpha}{2}} \) regularity estimates for such models, in the sense that

\[
\sup_{0 < r \leq r_0} \left( \frac{\inf_{\ell \in \mathcal{L}} \|u - \ell\|_{L^\infty(Q_r(x_0, t_0))}}{r^{1+\alpha}} \right) \leq \mathcal{C},
\]

where \( \mathcal{L} \) denotes the space of affine functions, \( \mathcal{C} > 0 \) is a constant depending only on dimension and structural parameters of operator and

\[
\alpha \in (0, \alpha_{\text{Hom}}) \cap \left( 0, 1 - \left( \frac{n}{p} + \frac{2}{q} \right) \right],
\]

where \( 0 < \alpha_{\text{Hom}} < 1 \) is the Hölder exponent comes from regularity for homogeneous problem with \( F \) having constant coefficients and without dependence on first order terms.

Such regularity issues play an essential role in a number of physical, geometric and free boundary problems, where obtaining a (quantitative) sharp regularity estimate for solutions is decisive for a finer analysis.

The mathematical insights in order to prove such sharp regularity estimates are based on a refined compactness method, as well as a systematic iterative approximation procedure arising from Caffarelli’s implementations for regularity theory to nonlinear equations and free boundary problems. These estimates are an extension/improvement to ones obtained by Michael G. Crandall \textit{et al}, Eduardo V. Teixeira and Lihe Wang in previous works.

This is joint work with Eduardo V. Teixeira (University of Central Florida - USA).

\textbf{Integral operators of fractional type and their commutators with Lipschitz}
symbols on weighted spaces, Estefanía Dalmasso, Gladis Pradolini and Wilfredo Ramos dafnedalm@gmail.com

We shall consider fractional integral operators of convolution type, $T_\alpha$ for $0 < \alpha < n$, given by

$$T_\alpha f(x) = \int_{\mathbb{R}^n} K_\alpha(x - y) f(y) dy,$$

where the kernel $K_\alpha$ is not identically zero and verifies the following conditions:

1. (Size) $K_\alpha \in S_\alpha$, that is, the inequality

$$\int_{s < |x| \leq 2s} |K_\alpha(x)| \, dx \leq C s^\alpha,$$

holds for every $s > 0$ and a positive constant $C$;

2. (Smoothness) $K_\alpha \in H^*_{\alpha, \infty}$, i.e., there exist a positive constant $C$ and $0 < \eta \leq 1$ such that

$$|K_\alpha(x - y) - K_\alpha(x' - y)| + |K_\alpha(y - x) - K_\alpha(y - x')| \leq C \frac{|x - x'|^\eta}{|x - y|^{n-\alpha+\eta}},$$

whenever $|x - y| \geq 2|x - x'|$.

We shall also analyze the case of their higher order commutators with symbol $b$, which satisfies a pointwise Lipschitz condition.

For all of the above operators, we will prove boundedness results on different weighted spaces, including $L^p-L^q$, $L^p-BMO$ and $L^p$–Lipschitz estimates. As far as we know, these results are new even in the unweighted case.

Afterwards, we will show that the smoothness condition can be weakened, being replaced by a Hörmander type condition in the scale of $L^p$ spaces or, more generally, in the scale of Orlicz spaces.

Finally, we will give a characterization result involving symbols of the commutators and continuity results for extreme values of $p$.

The Zakharov limit of the Zakharov-Rubenchik system, Juan C. Cordero Departamento de Matemáticas y Estadística, Universidad Nacional de Colombia, Manizales, Colombia. jccorderoc@unal.edu.co
We present weak and strong convergence results of solutions of the Zakharov-Rubenchik system when appropriate parameters go to zero. This limit solutions are known as the supersonic limit for the Zakharov-Rubenchik system. The proof of the weak limit is a classical argument in the theory of compactness, whose main ingredient is the Aubin-Lions Theorem and the Ascoli Theorem. Strong limits are conveniently treated by decomposing the nonlinearities and using the Strichartz estimates associated with the group of the Schrödinger equation and the wave group.

**Existence and uniqueness of combustion waves in porous media**, Grigori Chapiro *Universidade Federal de Juiz de Fora, Brazil* grigori@ice.ufjf.br

The method of in-situ combustion is a thermal technique with great potential for use in the exploration of oil. The modeling of combustion in porous media involves Fluid Dynamics and Chemical Kinetics. The models describing in-situ combustion are composed by reaction-convection-diffusion equations, present different scales (stiff problems) and are difficult to solve both mathematically and computationally. In this presentation some recent advances in the area will be shown. We study the combustion waves that occur when air is injected into a porous medium containing initially some solid fuel and prove the existence of traveling waves using phase plane analysis. We also identify all the possible ways that combustion waves and contact discontinuities can combine to produce wave sequences that solve boundary value problems with generic constant boundary data. The analysis is supported by numerical simulations. Stability of the combustion waves is also addressed.

**Asymptotic Density of Test Elements in Free Groups and Surface Groups**, Ilir Snopche and Slobodan Tanushevski *Universidade Federal do Rio de Janeiro - Rio de Janeiro - Brazil* ilirsnopche@gmail.com

An element $g$ of a group $G$ is called a test element if for any endomorphism $\varphi$ of $G$, $\varphi(g) = g$ implies that $\varphi$ is an automorphism. The first non-trivial example of a test element was given by Nielsen in 1918, when he proved that every endo-
morphism of a free group of rank 2 that fixes the commutator \([x_1, x_2]\) of a pair of generators must be an automorphism.

Let \(G\) be a finitely generated group with a finite generating set \(X\), \(d_X\) the word metric on \(G\) with respect to \(X\) and \(B_X(r)\) the ball of radius \(r \geq 0\) centered at the identity in the metric space \((G, d_X)\). Given \(S \subseteq G\), the asymptotic density of \(S\) in \(G\) with respect to \(X\) is defined as

\[
\overline{\rho}_X(S) = \limsup_{k \to \infty} \frac{|S \cap B_X(k)|}{|B_X(k)|}.
\]

In this short communication will be discussed the asymptotic density of test elements in free groups and surface groups.

**Nonlocal diffusion equations in homogeneous Lie groups**, Isolda E. Cardoso and Raúl E. Vidal  
isolda@fceia.unr.edu.ar

The asymptotic behaviour for several nonlocal diffusion models in the whole \(\mathbb{R}^n\) has been profusely studied. In certain cases, authors have related this behaviour with the one of the local diffusion model. Consider some \(u(x, t)\) that models the probabilist density function of a single population at the point \(x\) at time \(t\) and let \(J\) be a function such that \(\int_{\mathbb{R}^n} Jdx = 1\) where: \(J(x - y)\) is the probability distribution of jumping from location \(y\) to location \(x\), \(J \ast u(x, t)\) is the rate at which individuals are arriving to position \(x\) from all other places, and \(u(x, t) = \int_{\mathbb{R}^n} J(x - y)dyu(x, t)\) is the rate at which they are leaving location \(x\) to travel to all other sites. Then \(u\) satisfies a nonlocal evolution equation of the form \(u_t(x, y) = J \ast u(x, t) - u(x, t)\). Cortazar, Elgueta and Rossi (2009) studied the Dirichlet problem associated to this equation and proved that solutions of properly rescaled problems approximate uniformly the solution of the Dirichlet problem for the classical heat equation in \(\mathbb{R}^n\). Molino and Rossi (2016) also showed that the usual local evolution problems with spatial dependence can be approximated by non-local ones.

In this work we consider the same problems in the context of an homogeneous Lie group, in the sense of Folland and Stein. These groups are being recently brought to interest, specially the stratified ones, which we know as Carnot groups.
As an antecedent we refer to the work by Vidal (2017), who obtained similar results in the context of the Heisenberg group by means of the spherical transform related to the corresponding Gelfand pair.

**On the neighbourhoods of idempotents in zero-dimensional algebras**, Marcelo Sobottka *UFSC, Brazil* marcelo.sobottka@ufsc.br

A classic result due to Lev Pontryagin shows that for topological second-countable compact zero-dimensional groups, given any neighbourhood $V$ of the identity one can find a clopen neighbourhood of the identity $U \subset V$ which is a normal subgroup. In this work, we prove some versions of such result for other algebraic structures. More specifically we consider $(A, \cdot)$ a topological second-countable compact zero-dimensional algebra, that is, $A$ is a second-countable compact zero-dimensional topological space, and $\cdot$ a continuous binary operation in $A$. Thus, given any neighbourhood $V$ of an idempotent we are interested in finding $U \subset V$, a neighbourhood of this idempotent, such that $(U, \cdot)$ is a subalgebra. When $(A, \cdot)$ is a quasigroup, we will present sufficient conditions on the $\cdot$, under that such $U$ there exists and it is a clopen set. For the case when $(A, \cdot)$ is an inverse semigroup, we will prove that such $U$ there always exist and it is closed in $A$ and a relative open of the right (or left) class of the idempotent.

**Totally geodesic submanifolds of compact naturally reductive spaces**, Francisco Vittone franvittone@gmail.com

In this work we study the existence of non-flat totally geodesic submanifolds in compact naturally reductive homogeneous spaces.

The existence of non flat totally geodesic surfaces in symmetric spaces is well-known, and for the non-compact case, the the existence of a totally geodesic hyperbolic surface follows from Karpelevich’s theorem.

For a homogeneous space, the existence of homogeneous totally geodesic submanifolds of dimension or co-dimension equal to one has been widely studied, but little is known of the cases where the dimension of the submanifold is in between these two extreme cases.
The main result we will present, is that for a compact non flat Riemannian naturally reductive homogeneous space, there always exists a compact non flat Riemannian naturally reductive homogeneous space of dimension either 2 or 3 equivariantly and totally geodesic embedded in it.

This is a joint work with Antonio J. Di Scala and Carlos Olmos.

Bounding the nilpotency class of a finite p-group with a large normal abelian subgroup, Risto Atanasov ratanasov@email.wcu.edu

A finite $p$-group $G$ is called powerful if either $p$ is odd and $[G, G] \subseteq G^p$ or $p = 2$ and $[G, G] \subseteq G^4$. We will discuss results that bound the nilpotency class of powerful $p$-groups and $p$-central groups in terms of the exponent of a quotient by a normal abelian subgroup. This is a joint work with Ilir Snopche and Slobodan Tanushevski.

Finiteness and existence of attractors and repellers on sectional hyperbolic sets, Andres m. Lopez Barragan andresmlopezb@gmail.com

We study small perturbations of a sectional hyperbolic set of a vector field on a compact manifold. Indeed, we obtain an upper bound for the number of attractors and repellers that can arise from these perturbations. Moreover, no repeller can arise if the unperturbed set has singularities, is connected and consists of nonwandering points.

On previous results in dimension three (Arbieto, Morales) an upper bound in terms of the number of singularities is obtained but for transitive or nonwandering flows. Also, the author in higher dimensions obtain the same results for transitive flows. On the other hand there are examples of connected sectional hyperbolic sets (with or without singularities) containing nontrivial repellers (Anomalous Anosov flow). Thus, we will remove both transitivity and nonwandering hypotheses in order to obtain the finitude in a robust way of attractors and repellers for higher dimensional sectional hyperbolic sets. Moreover, we prove the non-existence of repellers for any perturbation of a connected sectional hyperbolic sets (with singularities) contained in the nonwandering set.
Main Theorem also provides a corollary related to one of the Bonatti’s conjectures. Bonatti stated a slightly stronger conjecture (from Palis), namely, generic diffeomorphisms that are far from homoclinic tangencies have only finitely many attractors and repellers. It is natural to consider even the flow version of Bonatti’s conjecture, namely, if a generic flow far from homoclinic tangencies has only a finite number of attractors and repellers. Thus, Main Theorem and the result announced by Crovisier-Yang immediately gives the flow version of Bonatti’s conjecture.

**Mean value properties, maximum principles and Harnack theorems for the solutions of the Hermite and Ornstein-Ulhembeck equations**, Guillermo J. Flores G. J. Flores, CIEM-FaMAF, Universidad Nacional de Córdoba, Av. Medina Allende s/n, Ciudad Universitaria, CP:X5000HUA Córdoba, Argentina guillexeneize@gmail.com

As a classical equation with long history, the heat equation is given by

\[ u_t = \Delta u, \]

while the Hermite and Ornstein-Ulhembeck equations associated with heat are given respectively by

\[ u_t = \Delta u - |x|^2 u \quad \text{and} \quad u_t = \Delta u - 2x \cdot \nabla u. \]

In this work we give **mean value properties, strong and weak maximum principles**, and **Harnack theorems** for the solutions of the latter equations, from a known explicit relation between the solutions of the heat equation and the solutions of the Hermite and the Ornstein-Ulhembeck equations. Thus, this allows us to obtain results about uniqueness and non-uniqueness, regularity, infinite propagation, Harnack Monotone Convergence, among others, for the solutions of the involved equations.

This talk is based on a joint work with Gustavo Garrigós.

In Thailand, the biomass derived fuel is an outstanding alternative energy due to agricultural products. Solid oxide fuel cell is a clean energy using hydrogen from water to produce the electric energy. However, to produce hydrogen for solid oxide fuel, other sources of energy are required. Up to now, there are plenty of researchers who develop the process of feeding biomass derived fuel to solid oxide fuel cell. In this work, a mathematical model of solid oxide fuel cell is presented with variety of equations that are used to predict the effect of solid oxide fuel cell fed biomass derived fuel. The results of the model are solved numerically and presented in three dimensional domain.

Stability of neutral type single neuron system with Lévy noise, G. Arthi PSGR Krishnammal College for Women, India arthimath@gmail.com

This work deals with the asymptotic stability for neutral type stochastic single neuron system. The noise term in the single neuron model is considered as Lévy type noise which consist of both Brownian motion and Poisson random measures. Using the Lyapunov-Krasovskii theory together with stochastic analysis, the delay-dependent sufficient conditions to ensure that the stability of the system are derived in terms of linear matrix inequality. Finally, numerical examples are provided to validate the effectiveness of the proposed results.

Numerical modelling of rheologically complex fluids including pressure correction method - Relevance to atherosclerosis, KM Surabhi and D. Srikanth Department of Applied Mathematics, Defence Institute of Advanced Technology, Pune - 411025, India and Department of Applied Mathematics, Defence Institute of Advanced Technology, Pune - 411025, India surabhi_phdam15@diat.ac.in

In this presentation explores, cardiac catheterization, an invasive diagnostic procedure of the atherosclerosis. In this surgical technique, drug is transported to
the targeted region through the intervention of the catheter into the blood vessel. Considering the drug coated with the nano particles on the outer surface of the fully immersed catheter into the flow regime. Mathematical model of the blood-nano fluid flow through stenotic artery in the presence of catheter, is analysed in detail. Blood is rheologically complex fluid, considered as non-Newtonian fluid. Presence of the nano particles on the outer layer of catheter, in this manner, investigate the temperature and concentration distribution in the targeted region. In the pumping action of the heart, artery wall is considered to be flexible in nature, which is accomplished by acquainting the time variable. The concluded mathematical model, governed by coupled non-linear equations and these equations are solved by using the numerical methods. In solving the momentum equations with the pressure by using the marker and cell (MAC) method developed by Harlow and Welch (1965) for incompressible unsteady viscous fluid, is a pressure correction method. Poisson’s equation of pressure is solved by according the successive over relaxation (SOR) method. Consequently, evaluating the effects of fluid and the embedded geometric parameters on the hemodynamic characteristics such as velocity profiles, pressure drop, flow resistance and wall shear stress. Also, enquired the impact of the non-dimensional numbers, which are accommodate in the non-dimensionalization form of mathematical model, on the hemodynamics of the flow as well as on the temperature and concentration profiles. This physical model is applicable to the pharmaceutical industry and Biomedical community.

Closed minimal hypersurfaces of the 5-dimensional sphere with constant scalar curvature and constant third symmetric function, L. A. M. Sousa Jr. 
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Let \( \mathcal{F}_4 \) be the family of all closed minimally immersed hypersurfaces with constant scalar curvature \( R \) in the unit sphere \( S^5 \). Let \( \lambda_1, \ldots, \lambda_4 \) be the principal curvatures of \( M^4 \in \mathcal{F}_4 \). The third symmetric function \( f_3 \) is defined by \( f_3 = \sum_{i=1}^{4} \lambda_i^3 \). Recently Deng, Gu and Wei proved that if \( M^4 \in \mathcal{F}_4 \) has vanishing function \( f_3 \) then \( M^4 \) is isoparametric, i.e, \( M^4 \) has constant principal curvatures. In this paper
we prove that if $M^4 \in \mathfrak{F}_4$ has non-zero constant $f_3$ and $R \geq 0$ then $M^4$ is isometric to a Clifford torus $S^1 \left( \frac{1}{2} \right) \times S^3 \left( \frac{\sqrt{3}}{2} \right)$. Combining this result with Deng-Gu-Wei theorem we obtain the classification of hypersurfaces $M^4 \in \mathfrak{F}_4$ with constant symmetric function $f_3$ and non-negative scalar curvature $R$, which is directly related to the following conjecture posed by S.S. Chern 50 years ago and that remains open for $n \geq 4$.

**Chern Conjecture:** Let $M^n, n \geq 3$, be a closed minimal hypersurface of $S^{n+1}$ with constant scalar curvature $R$ and let $R_n$ be the set of possible values for $R$. Is $R_n$ a discrete set of real numbers?

**Weak solvability and optimal control for a class of partial integro-differential equations**, Andrii V. Anikushyn

*Taras Shevchenko National University of Kyiv, Ukraine*

We consider a class of linear partial integro-differential equations of Volterra-type that generalizes the heat equation. It corresponds to situations when history of the described process is taking into account. For this class we provide a priori estimates in appropriate Hilbert spaces. Based on them we prove well-posedness of the corresponding boundary-value problem. Further, we consider an optimal control problem. Control function is in the right hand side of the equation and belongs to negative spaces (e.g. combination of Dirac delta functions). Using a priori inequalities we show the existence of an optimal control problem for appropriate quality criterium.

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**Positive definiteness and integral representations of special functions**, Jorge C. Buescu and António C. Paixão

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We characterize a holomorphic positive definite function $f$ defined on a horizontal strip of the complex plane as the Fourier-Laplace transform of a unique exponentially finite measure on $\mathbb{R}$. The classical theorems of Bochner on positive definite functions and of Widder on exponentially convex functions become
special cases of this characterization: they are respectively the real and pure imaginary sections of the complex integral representation. We apply this integral representation to special cases, including the \( \Gamma \), \( \zeta \) and Bessel functions, and construct explicitly the corresponding measures, thus providing new insight into the nature of complex positive and co-positive definite functions: in the case of the zeta function this process leads to a new proof of an integral representation on the critical strip.

**Weak solutions for a nonlocal singular elliptic problem**, Moloud Makvand Chaharlang and Abdolrahman Razani  
[Email](mailto:moloudmakvand@gmail.com)

In this article, a nonlocal fourth order singular elliptic problem is considered where the exponent in the nonlocal term is different from that in the singular term. The existence of infinitely many solutions is proved by using the variational methods in Sobolev Spaces and the critical points principle of Ricceri.

**Source-dependent noise removal using spectral graph theoretic approach**, Pacheeri Padikkal Jidesh and A. A. Bini  
[Email](mailto:ppjidesh@gmail.com)

In this study we propose to design a model suited for denoising images or signals corrupted with data-dependent noise which follows a Gamma or a Poisson distribution. The spectral graph theory serves as a backbone of this work, where we design a Graph Laplacian operator which performs a controlled diffusion in the image. The image being an undirected weighted graph with connected components, its Laplacian matrix is symmetric and positive semi-definite and has an Eigen value decomposition. The Laplacian operator forms a closed form solution to the parabolic heat diffusion. The diffusion magnitude can be controlled by non-local weights based on a radiometric/photometric and a spatiometric distances corresponding to the pixel values and their locations, respectively. The overall procedure involves design of a graph from the input image and extract the graph Laplacian from the graph. The weight matrix which controls the magnitude of diffusion is carefully designed so as to care the intensity variations in the image due to edges and other important details. A variational formulation is designed
to minimize the non-local graph Laplacian in order to design a functional with constraints derived from the MAP estimate of the noise distribution under consideration. The model converges to a unique solution as the functional so designed is convex. Since the convergence depends on the time step used for solving the iterative scheme as well as the regularizing parameter, a parameter insensitive model based on split-Bregman formulation is being employed for solving the same. This reformulation under the split-Bregman scheme makes the convergence faster and neutralizes the effect of parameter on the convergence process.

**Some results in distal theories**, Gareth Boxall and Charlotte Kestner

Distal theories were conceived as being those NIP theories ‘furthest’ away from being stable. I will give a brief introduction to distal theories, highlighting interesting equivalences in their definitions. I will then go on to discuss some results in distal theories. In particular I will discuss the definable \((p, q)\)-theorem, which can be seen as a strengthening (in this context) of the \((p, q)\) – theorem from statistical learning theory. This is joint work with G. Boxall.

**Math in the Sky: Using Alexander Graham Bell’s kites to raise public awareness of mathematics**, João J. Queiroz and Humberto J. Bortolossi

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At the beginning of the 20th century, one of the issues confronting scientists at the time was the possibility of building large and stable flying devices to take a man to the sky and bring him back safely. Alexander Graham Bell proposed a flying machine – a kite – that, in fact, managed to transport a man. The idea of Bell: to use regular tetrahedrons as cells of the structures of his kites. Taking this historical episode as background, we have devised a workshop to be held in Math festivals and schools aiming to raise public awareness of Mathematics. The subject, besides interesting from its historical perspective, is very opportune to explore counting, similarity, proportionality, areas, and volumes related to the juxtaposition of tetrahedrons, and the Galileo Galilei’s principle of similitude. There is a complete set of materials to assist in conducting the workshop: slides with the narrative, a de-
Detailed guide showing how to build the Alexander Graham Bell’s tetrahedral kites with a low-cost material, a worksheet with suggestions of mathematical questions and a set with 3D computer models. We’ve conducted this workshop several times in festivals, workshops in schools and teacher professional development courses (https://www.youtube.com/watch?v=C_a7nVVCeTE).

**Toric topology of the complex Grassmann manifolds**, Victor M. Buchstaber and Svjetlana Terzic Steklov Mathematical Institute, Russian Academy of Sciences, Gubkina Street 8, 119991 Moscow, Russia and Faculty of Science, University of Montenegro, Džordža Vašingtona bb, 81000 Podgorica, Montenegro buchstab@mi.ras.ru

The family of complex Grassmann manifolds $G_{n,k}$ with the canonical action of the algebraic torus $(\mathbb{C}^*)^n$ and, consequently, the compact torus $T^n = \mathbb{T}^n$ is well known. The interest in description of these actions is motivated by the classical and modern problems of algebraic geometry, algebraic and equivariant topology, symplectic geometry and enumerative combinatorics. In the well known papers of Gel’fand-Serganova, Goresky-MacPherson, Kapranov etc, it is studied the equivariant topology of $G_{n,k}$, based on the action of the algebraic torus $(\mathbb{C}^*)^n$. In this context, there is the analogous of the moment map $\mu : G_{n,k} \to \Delta_{n,k}$ for the hypersimplex $\Delta_{n,k}$. In the case $k = 1$ we have the complex projective space $\mathbb{C}P^{n-1}$, which is the fundamental example of a toric manifold and $\Delta_{n,1}$ is a simplex. For $k \geq 2$ the combinatorics of $\Delta_{n,k}$ does not determine the structure of the orbit space $G_{n,k}/T^n$.

We study the action of the compact torus $T^n$ on $G_{n,k}$ by developing the methods of the toric geometry and the toric topology and propose the method for the description of the orbit space $G_{n,k}/T^n$. The existence of the symmetric group $S_n$-action on $G_{n,k}$ simplifies the application of this method. Appealing on the structures we introduce for developing our method, we describe all regular and singular points of the moment map $\mu$. In our previous paper (1) we proved that $G_{4,2}/T^4$, which is the space of complexity 1, is homeomorphic to $\partial \Delta_{4,2} \ast \mathbb{C}P^1$. In this talk it will be presented our approach to this problem and the new methods for obtaining the results. As an application, it is obtained the explicit description of the orbit space $G_{5,2}/T^5$, which is the non-trivial example of the space of
complexity 2, and prove that it is homotopy equivalent to $\partial \Delta_{5,2} \ast \mathbb{C}P^2$.

The methods and the results, which aim to be presented, are very important for the construction of the theory of $(2l, q)$-manifolds we recently have been developing and which is concerned with $M^{2l}$-manifolds with an effective action of the torus $T^q$, $q \leq l$ and an analogous of the moment map $\mu : M^{2l} \to P^q$, where $P^q$ is a $q$-dimensional convex polytope.

Victor M. Buchstaber and Svjetlana Terzić, Topology and geometry of the canonical action of $T^4$ on the complex Grassmannian $G_{4,2}$ and the complex projective space $\mathbb{C}P^5$, Moscow Math. Jour. 16 (2), (2016), 237–273.

**Generalization of Exner models for bedload sediment transport in shallow water**, M.J. Castro Díaz, C. Escalante, E.D. Fernández Nieto, T. Morales de Luna and G. Narbona Reina Universidad de Córdoba, Spain tomas.morales@uco.es

Sediment can be transported in several ways by the action of a river. During low transport stages, particles move by sliding and rolling over the surface of the bed. This type of transport is usually called bedload transport. The usual approach to model these phenomena is to use the Saint-Venant-Exner model (SVE) which consists in a shallow water model coupled with a morphodynamical component for the bedload transport. The bedload transport depends on an empirical flux. Nevertheless, this approach presents some drawbacks, for instance, gravitational effects for bedload transport is neglected and the momentum equation for the sediment is missing.

Here we will present recent advances in modeling of bedload transport. In particular we will introduce a generalization of SVE models. The new model is composed of two layers: an upper layer consisting in clear water and a lower layer which accounts for the sediment material. Opposed to classical SVE, this models includes both mass and momentum equations for water and sediment. This allows to better describe the phenomena. The key point is the definition of the friction laws between the two layers. The model is a generalization of classic models as it allows to recover SVE system when the ratio between the hydrodynamic and morphodynamic time scales is small, as commonly done to derive SVE models. Moreover, the new model allows to describe bedload sediment transport for strong and weak interactions between the fluid and the sediment.
The model may be also extended by including a non-hydrostatic pressure correction for the fluid layer and the boundary condition at the sediment surface. Numerical simulations show that the model provides promising results and behave well in low transport rate regimes as well as in many other situations.

The two diophantine equations: \( A^4 + KB^4 = C^2 + k^2 D^4 \) and \( X^4 + 2Y^4 = Z^4 + 4W^4 \), Susil K. Jena susil_kumar@yahoo.co.uk

The Diophantine equation \( X^4 + 2Y^4 = Z^4 + 4W^4 \)

was suggested by Sir Peter Swinnerton-Dyer. It is related to a famous unsolved problem in the arithmetic of \( K3 \) surfaces. There is only one known solution of this equation given as:

\[
1484801^4 + 2 \times 1203120^4 = 1169407^4 + 4 \times 1157520^4.
\]

In this paper, we give a parametric solution of a closely related Diophantine equation

\[
A^4 + kB^4 = C^2 + k^2 D^4,
\]

where \( k \) is any non-zero integer. This parametric solution allows us to calculate any number of non-trivial and primitive integer solutions for \( (A, B, C, D) \) for any fixed non-zero integer \( k \). The solutions are non-trivial, if \( kABCD \neq 0 \); and primitive, if \( \gcd(A, kB, C, kD) = 1 \). For \( k = 2 \), we calculate many such solutions of this latter equation of which one solution is given as:

\[
621^4 + 2 \times 250^4 = 395609^2 + 4 \times 50^4.
\]

Further, with \( k = 2 \), we explore the possibility of making \( C \) a square so that the problem proposed by Swinnerton-Dyer may be settled.

The Principle of Ariadne as a New Choice Paradigm in Mathematics, Walter A. Carnielli walter.carnielli@gmail.com
Mathematics, as any other intellectual discipline, needs principles. The Axiom of Choice (AC) is one of the most important, yet the most controversial, of such principles. Certain parts of set-theoretic and mathematical investigation simply do not make sense in AC contexts, and claim for new principles, which may lead to surprisingly deep new mathematical realms. One famous principle of this kind is the Axiom of Determinacy (AD), which contradicts AC while leading to a new world where every set has the property of Baire and every set of reals is Lebesgue measurable.

The Principle of Ariadne (formulated in 1988 by W. Carnielli and C. Di Prisco and later published in 1993) is a new infinitary choice principle that is proved to be independent of the AC in ZF, although it can be consistently added to the remaining ZF axioms. The present talk surveys, and motivates, the foundational importance of the Principle of Ariadne and proposes the Ariadne Game, showing that the Principle of Ariadne, corresponds precisely to a winning strategy for the Ariadne Game. Some relations to other alternative set-theoretical principles, AD among them, will be also briefly discussed.

Invariants of The Weyl Algebra: Noncommutative Noether’s Problem and Gelfand-Tsetlin Categories, João Schwarz, Vyacheslav Futorny and Farkhod Eshmatov jfschwarz.0791@gmail.com

We discuss some natural noncommutative versions of the Noether’s Problem on rings of differential operators. We follow Gelfand-Kirillov philosophy that Weyl fields, that is, the total quotient rings of the Weyl algebras, are important noncommutative analogues to the field of rational functions. We prove that the invariants of the Weyl fields under the action of an unitary reflection group are isomorphic to the original Weyl field. We also prove a similar statement for the total ring of quotients of differential operators on the torus under certain actions of the classical Weyl groups. As applications we prove a Gelfand-Kirillov theorem for rational Cherednik algebras and certain Galois algebras. We discuss how this relates to Gelfand-Tsetlin modules and the Gelfand-Kirillov Conjecture for the classical Lie algebras. Finally, we exhibit a Gelfand-Tsetlin category of modules for symmetric differential operators.
Construction of spherical codes using the Hopf fibration, Henrique K. Miyamoto, Henrique N. Sá Earp and Sueli I. Costa miyamotohk@gmail.com

A spherical code is a finite set of points on the surface of a unit sphere \( S^{n-1} \subset \mathbb{R}^n \). The problem of placing points on a sphere of a given dimension has relevance to diverse fields. In communication theory, point sets on a sphere are useful for transmitting over a Gaussian channel and are a natural generalisation of phase shift keying (PSK) signal sets to dimensions greater than two. The spherical code problem in this case is formulated as a packing problem, which we have addressed: given a minimum distance \( d \), to find the largest possible number of points \( M \) on \( S^{n-1} \) such that the Euclidean distance between any two of them is at least \( d \). A new approach to construct a family of spherical codes is presented, based on properties of the Hopf fibration and inspired by a previous TLSC (torus layers spherical codes) construction by Torezzan et al. (IEEE Trans. Inf. Theory, 2013). In the basic case (\( \mathbb{R}^4 \)), we use the Hopf foliation of the 3-sphere by tori to construct a two-step algorithm: (i) to choose a torus parametrised by height and (ii) to distribute points in each torus by iterated rotation matrices. In this dimension, our construction is analogous to the previous method and matches its performance. The procedure can be extended to dimensions \( 2^n \) by considering selected \( S^{n-1}_{\sin \eta_i} \times S^{n-1}_{\cos \eta_i} \) in the foliation of \( S^{2n-1} \) by manifolds \( S^{n-1}_{\sin \eta} \times S^{n-1}_{\cos \eta} (n = 4, 8, 16...) \) and in each of these spheres \( S^{n-1}_{\sin \eta} \) and \( S^{n-1}_{\cos \eta} \) recursively distribute points at a scaled minimum distance. In these higher dimensions, for some minimum distance, our procedure has better performance in comparison with previous TLSC implementations for some minimum distances \( d \). The spherical code construction proposed has some advantage due to its algorithmic procedures for coding and decoding processes.

MHD mixed convection flow in a lid-driven cavity involving a solid body, Canan Bozkaya bcanan@metu.edu.tr

The two-dimensional, steady, laminar mixed convection flow and the heat transfer characteristics are numerically investigated in a lid-driven square cavity with a solid body in the presence of a horizontally applied uniform magnetic field. The stationary horizontal walls of the cavity are considered to be adiabatic
whereas the moving vertical walls are kept cold. A solid square block which has a thermal conductivity and generates heat is placed inside the cavity. The heat generation results in an additional heat conduction equation for the solid body in the system of governing equations. In the present study, the stream function-vorticity-temperature formulation of mixed convection flow is followed, and the equations are discretized using the dual reciprocity boundary element method (DRBEM) with constant elements. The DRBEM aims to transform the differential equations into equivalent integral equations only on the boundary of the computational domain by treating the non-homogeneity through a radial basis function approximation. Numerical simulations are carried out for various controlling parameters such as Richardson number, Hartmann number and joule heating parameter as well as the location of the solid body. The obtained results show that the flow behavior and the heat transfer rate are significantly influenced by the use of different combination of aforementioned parameters in the presence of a solid body. Moreover, according to the location of the solid body, the size of the eddies in fluid flow and the distribution of the isotherms drastically vary, which indicates that the solid body can be used to control the heat transfer enhancement and the flow characteristics in a cavity subject to an external magnetic field.

Lagrangian fibrations by Prym varieties, Justin Sawon sawon@email.unc.edu

The Hitchin systems are Lagrangian fibrations on moduli spaces of Higgs bundles. Their compact counterparts are Lagrangian fibrations on compact holomorphic symplectic manifolds, such as the integrable systems of Beauville-Mukai, Debarre, Arbarello-Ferretti-Saccà, Markushevich-Tikhomirov, and Matteini. The GL-Hitchin system and the Beauville-Mukai system are both fibrations by Jacobians of curves. Thus they are isomorphic to their own dual fibrations. Moreover, Donagi-Ein-Lazarsfeld showed that the Beauville-Mukai system can be degenerated to a compactification of the GL-Hitchin system. The other Hitchin systems and the other compact integrable systems mentioned above are fibrations by Prym varieties. In this work, we explore the relations between these different
Lagrangian fibrations. In particular, we describe degenerations of the compact examples to compactifications of SL-, PGL-, SO-, and Sp-Hitchin systems. We also describe dual fibrations of certain Lagrangian fibrations by Prym varieties.

**Prolongation of solutions of measure differential equations and dynamic equations on time scales**, Márcia Federson, Rogélio Grau and Jaqueline Godoy Mesquita *Universidade de São Paulo, Brazil, Universidad del Norte, Colombia and University of Brasília, Brazil* federson@icmc.usp.br, rogeliograu@gmail.com and jgmesquita.unb@gmail.com

In this talk, we prove the results on existence and uniqueness of the maximal solutions for measure differential equations, considering more general conditions on functions $f$ and $g$ by using the correspondence between the solutions of these equations and the solutions of generalized ODEs. Moreover, we prove these results for the dynamic equations on time scales, using the correspondence between the solutions of these last equations and the solutions of the measure differential equations.

**Reconstruction for Diffusion Maps**, Francisco D. Moura Neto, Ricardo Fabbri and Lúcia Pinto fdmouraneto@gmail.com

Diffusion maps is a dimensionality reduction technique based on a Markov chain defined on a similarity graph of a data set. Several research papers have been published following the pioneering work of R.R. Coifman, S. Lafon (Diffusion maps, Appl. Comput. Harmon. Anal. 21 (2006) 5–30). One less investigated subject is the reconstruction of a datapoint from its reduced dimensional representation. We present a datadriven reconstruction procedure based on a Tikhonov regularization. This procedure is applied not only to diffusion maps but also to Principal Components Analysis. In the latter case, the resulting reconstruction can be non-linear, depending on the data set, in which case supersedes the usual linear reconstruction approach. For diffusion maps reconstruction, the proposed procedure keeps the error within acceptable bounds. We apply the procedure in the reconstruction of letters from a low dimensional representation.
Motivic zeta functions, orbifold motivic measures and Q-resolutions of singularities, Edwin León-Cardenal, Jorge Martín-Morales and Juan Viu-Sos Centro de Investigación en Matemáticas (CIMAT), Mexico, Centro Universitario de la Defensa, IUMA, Spain and Universidade de São Paulo, Brazil edwin.leon@cimat.mx, jorge@unizar.es and jviusos@math.cnrs.fr

The motivic zeta function $Z_{\text{mot}}(f; s)$ is a geometrical invariant associated to a complex polynomial $f \in \mathbb{C}[x_1, \ldots, x_n]$, introduced by Denef and Loeser in 1998 as a generalization of the topological zeta function $Z_{\text{top}}(f; s)$ and the Igusa’s $p$-adic zeta function of $f$ by using Kontsevich’s motivic integration theory.

The previous functions are classically computed in terms of an embedded resolution of singularities of $f^{-1}(0) \subset \mathbb{A}^n_{\mathbb{C}}$, where every exceptional divisor gives a “pole candidate” $s_0$ for $Z_{\text{mot}}(f; s)$ (or $Z_{\text{top}}(f; s)$), which could be not a real pole when one gets the final expression.

The Monodromy Conjecture affirms that any pole $s_0$ gives an eigenvalue $\exp(2\pi s_0)$ of the monodromy on the cohomology of the Milnor fiber of $f^{-1}(0)$.

The latter is proved in some particular cases, but one of the main difficulties to approach this conjecture is the fact that minimal resolutions of singularities does not exist for $n > 2$, the resolution models are complicated to compute and could become very complexes in terms of number of exceptional divisors and relations between them, providing a lot of “bad pole candidates”.

In this work, we study the motivic zeta function $Z_{\text{mot}}(f; s)$ (and its specialization in $Z_{\text{top}}(f; s)$) in terms of the so-called embedded $\mathbb{Q}$-resolutions of singularities of $f^{-1}(0)$, which are roughly embedded resolutions $\pi : X \to \mathbb{C}^n$ where the ambient space $X$ is allowed to contain abelian quotient singularities, providing a “simpler” model with less exceptional divisors and thus less “bad pole candidates” for $Z_{\text{mot}}(f; s)$.

Error estimates and adaptivity in mimetic schemes for bidimensional diffusion-convective problems, Giovanni E. Calderón, Carlos F. Torres, Jorge Villamizar and Julio C. Carrillo Universidad Industrial de Santander, Colombia and Universidad de Los Andes, Venezuela gcalderon@matematicas.uis.edu.co, jorge@matematicas.uis.edu.co and dctorres@ula.ve
In recent years, mimetic schemes are increasingly present in the numerical resolution of science and engineering problems, being for the present completely unknown the adaptive processes for this type of numerical method. In this work, we present a first adaptive process that defines an optimal mesh to calculate the solution of bidimensional convection diffusion problems. The error estimate is made from the discrete version of the gradient operator. The numerical experimentation shows the good behavior of the procedure.

High-density hard-sphere lattice configurations in 2D and 3D, Alexander Mazel, Izabella Stuhl and Yuri Suhov alik.mazel@gmail.com, stuhlizabella@gmail.comandyms@statslab.cam.ac.uk

We study random high-density hard-sphere configurations on a 2D triangular lattice and a 3D FCC lattice (A2 and A3), for a given attainable exclusion diameter $D$. In 2D, assuming certain arithmetic properties of $D$ (related to Eisenstein primes), we identify the extreme periodic Gibbs measures for large fugacities and analyze their properties. This measures are generated by ground states – dense-packing circle configurations (inclined or non-inclined $D$-sub-lattices and their space-shifts). In 3D we analyze a few lower values of $D$: here the ground states are connected to $D$-sub-lattices (FCC dense-packed sphere configurations) and HCP dense-packed configurations. The structure of Gibbs measures is different for various values of the exclusion diameter $D$.

The theoretical basis is provided by the Pirogov–Sinai theory complemented with the theory of dominant ground states. Some of our proofs are computer assisted.

On $\mathbb{Z}_d$-symmetry of spectra of linear operators, Oleg Reinov Saint Petersburg State University, Russia orein51@mail.ru

It was shown by M. I. Zelikin (2008) that the spectrum of a nuclear operator in a separable Hilbert space is central-symmetric iff the traces of all odd powers of the operator equal zero. The criterium can not be extended to the case of general Banach spaces: It follows from Grothendieck-Enflo results that there exists
a nuclear operator $U$ in the space $l_1$ with the property that $\text{trace } U = 1$ and $U^2 = 0$.

B. Mityagin (2016) generalized Zelikin’s criterium to the case of compact operators in Banach spaces some of which powers are nuclear (he considered even the so-called $\mathbb{Z}_d$-symmetry of spectra).

We give sharp generalizations of Zelikin’s theorem and of Mityagin’s result (for the case where the operators are not necessarily compact). Our results are optimal. In particular, we have the following (sharp) generalization of Grothendieck-Enflo theorem.

**Theorem.** There exist an asymptotically Hilbertian space $Y_2 := \left( \sum_k \frac{I^{n_k}}{q_k} \right)_{l_2}$ ($q_k \to 2$ and $n_k \to \infty$) and a nuclear operator $U$ in this space so that

1) $U$ is $(1, 2 + \varepsilon)$-nuclear for each $\varepsilon > 0$.
2) $U$ is not $(1, 2)$-nuclear.
3) $\text{trace } U = 1$ and $U^2 = 0$.

The corresponding statements hold for the adjoint operator $U^*$.

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**Matroidal Structures for Covering based Rough Sets**, Mauricio Restrepo

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Covering based rough sets is an extension of Pawlak’s rough set theory, proposed for applying to more general contexts of information systems. In this case a covering is used instead of the partition obtained from an equivalence relation. Recently many authors introduce relationships between covering based rough sets, matroids and submodular functions. In this work, we present the matroidal structures obtained from different partitions and coverings of a specific set. We establish a partial order relation among the matroidal structures via submodular functions, induced from some coverings.

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**Generalized Widom–Rowlinson models and their applications**, Yuri Suhov, Alexander Mazel, Izabella Stuhl and Stefan Zohren

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We consider a class of models in Statistical Mechanics generalizing the Widom–Rowlinson model. Our models are set on a triangular lattice in 2D and
an FCC lattice in 3D. There are $q$ types of particles/cells characterized by (Euclidean) exclusion distances $D(i, j)$, $1 \leq i, j \leq q$. We study the structure of extreme periodic Gibbs measures and their connections with the ground states. Based on the theoretical results, a Markov process is constructed, in the space of admissible configurations, where cells can migrate, mutate, divide and die.

Numerical simulations have been conducted, and their results used in a number of applied domains, including Theoretical Biology, particularly tumor growth modelling.

**Equivariant simple singularities and their classification**, Evgeny Astashov
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Given two linear representations of a group $G$ on $\mathbb{C}^n$ and on $\mathbb{C}$, we call a function $f : \mathbb{C}^n \to \mathbb{C}$ *equivariant* if for all $\sigma \in G$, $z \in \mathbb{C}^n$ the condition $f(\sigma \cdot z) = \sigma \cdot f(z)$ holds. A similar definition can be given for automorphisms of $\mathbb{C}^n$.

The group $D_{n}^{GG}$ of equivariant biholomorphic germs $\Phi : (\mathbb{C}^n, 0) \to (\mathbb{C}^n, 0)$ acts on the space $O_{n}^{GG}$ of equivariant holomorphic function germs $f : (\mathbb{C}^n, 0) \to (\mathbb{C}, 0)$. This space is split into orbits of this action, and so are spaces $j_r O_{n}^{GG}$ of $r$-jets at 0 of germs from $O_{n}^{GG}$. An orbit $D_{n}^{GG}(j_r g) \subset j_r O_{n}^{GG}$ is said to be adjacent to the orbit $D_{n}^{GG}(j_r f)$ if any neighborhood of some point in $D_{n}^{GG}(j_r f)$ intersects $D_{n}^{GG}(j_r g)$.

A germ $f \in O_{n}^{GG}$ is called *equivariant simple* if for all $r \in \mathbb{N}$ the orbit $D_{n}^{GG}(j_r f) \subset j_r O_{n}^{GG}$ has a finite number of adjacent orbits, and this number is bounded from above by a constant independent of $r$.

Two germs $f, g \in O_{n}^{GG}$ are called *equivariant right equivalent* if there exists a germ $\Phi \in D_{n}^{GG}$ such that $g = f \circ \Phi$.

There exists a general problem to classify equivariant simple function germs in $O_{n}^{GG}$ with a critical point at $0 \in \mathbb{C}^n$ up to the above equivalence relation (for a given abelian group $G$ and a pair of its representations on the source and target). This problem generalizes a similar one for the non-equivariant case solved by V. I. Arnold in 1972 (the result is known as ADE-singularities).

In the talk I will present some recent results related to this problem. In particular, a complete classification of equivariant simple singularities in 2 and 3 variables will be presented for $G = \mathbb{Z}_3$. 

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The Localized Regular Dual Reciprocity Method (LRDRM) for PDE initial value problems, Nahuel Caruso and Margarita Portapila caruso@cifasis-conicet.gov.ar

The Localized Regular Dual Reciprocity Method (LRDRM) is an integral domain decomposition method that uses a Radial Basis Functions (RBF) interpolation with two distinguishing features, the boundary conditions are imposed at the local interpolation level and all the calculated integrals are regular. In addition an enhancement of the LRDRM enforces the interpolation functions to satisfy the stationary Partial Differential Equation to be solved. Different kernels can be considered in order to eliminate the contribution of the single layer potential.

In this work we present an extension of the LRDRM for non-stationary PDE initial value problems. Different time schemes are considered. The new formulation is tasted for time-dependent convection-diffusion equations. The numerical solutions show an accurate behaviour of the method for these problems.

On asymptotic behavior of blow-up solutions to nonlinear higher order ordinary differential equations, Irina Astashova ast@diffiety.ac.ru

We discuss the asymptotic behavior of blow-up solutions to the nonlinear higher order ordinary differential equations

\[ y^{(n)} = p(x, y, y', \ldots, y^{(n-1)}) |y|^k \text{sgn} y, \]  

(9)

where \( n \geq 1, \) real (not necessary natural) \( k > 1, \) and a continuous function \( p(x, y_0, \ldots, y_{n-1}) \) is Lipschitz continuous in the last \( n \) variables. I.Kiguradze (1990) conjectured the power-law asymptotic behavior of all such solutions. The conjecture seems natural because of the power upper and lower estimates obtained for all such solutions (I.Kiguradze, G.Kvinikadze). It was proved for \( n = 2 \) by I.Kiguradze and for \( n = 3, 4 \) by I.Astashova that all blow-up solutions have the power-law asymptotic behavior. Later the result was obtained on the existence of solutions with non-power-law asymptotic behavior for equation (9) with \( p = p_0 > 0, n = 12, 13, 14 \) (I.Astashova). V.Kozlov proved earlier the result for such type of equations with arbitrarily large \( n, \) but it was not clear how small can be \( n. \) Now it appears that the following weaker version of Kiguradze’s conjecture is
Theorem. Suppose $p \in C(\mathbb{R}^{n+1}) \cap Lip_{y_0,...,y_{n-1}}(\mathbb{R}^n)$ and $p \to p_0 > 0$ as $x \to x^*$, $y_0 \to \infty$, ..., $y_{n-1} \to \infty$. Then for any integer $n > 4$ there exists $K > 1$ such that for any real $k \in (1, K)$, any solution to equation (9) tending to $+\infty$ as $x \to x^* - 0$ has the power-law asymptotic behavior, namely

$$y(x) = C(x^* - x)^{-\alpha(1 + o(1))}$$

with

$$\alpha = \frac{n}{k - 1}, \quad C^{k-1} = \frac{1}{p_0} \prod_{j=0}^{n-1} (j + \alpha).$$

It is also proved that the typical behavior of blow-up solutions even to equation (9) with $p = p_0 > 0$ is not power-law one.

Lagrangian Floer homology of symplectic blow ups, Andrés Pedroza Universidad de Colima, Mexico andres_pedroza@ucol.mx

Hamiltonian and Lagragian Floer homology are powerfull tools developed by A. Floer to solve the Arnol’d Conjecture about the minimal number of fixed points of a Hamiltonian diffeomorphism. Nevertheless, Lagrangian Floer homology is important in its own right due to its rich algebraic structure and for its implication in the classification of Lagrangian embeddings on a symplectic manifold. Unfortunately, it is complicated to compute the Lagrangian Floer homology of a pair of Lagrangian submanifolds of a given symplectic manifold.

To that end, we show how to compute the Lagrangian Floer homology in the symplectic one-point blow up from the Lagrangian Floer homology in the base manifold. Furthermore, we restrict our attention to the case of Lagrangian Floer homology for monotone Lagrangian submanifolds as defined by Y.-G. Oh.

In particular we will focus on $(\mathbb{C}P^2, \omega)$ and its monotone one-point blow up. Here we show how $\mathbb{R}P^2$ induces a pair of Lagrangian submanifolds in the blow up that are not Hamiltonian isotopic, but are Hamiltonian isotopic in $(\mathbb{C}P^2, \omega)$.

Hamiltonian properties of line graphs and their generalizations, Zdenek Ry-
We consider line graphs and, as generalizations, the classes of claw-free (i.e., $K_{1,3}$-free) graphs, $\{K_{1,4}, K_{1,4} + e\}$-free graphs and line graphs of 3-hypergraphs. It is known that many results on hamiltonian properties of line graphs can be directly extended to claw-free graphs using the closure operation that turns a claw-free graph into a line graph of a triangle-free graph while preserving (many of) its Hamilton-type properties. We generalize the closure and use it as a tool to extend some results to the above mentioned superclasses of line graphs. Special attention will be paid to the famous conjectures by Thomassen (every 4-connected line graph is hamiltonian) and by Matthews and Sumner (every 4-connected claw-free graph is hamiltonian). These conjectures were shown to be equivalent by an application of the closure, and we show their seemingly much stronger, but still equivalent versions on hamiltonicity, 1-Hamilton-connectedness, Tutte cycles etc. in the above mentioned superclasses of line graphs. We also discuss some computational complexity consequences and present recent best known positive results in the direction of the conjectures.

**The Schrödinger-Boussinesq system on weak $L^r$ spaces**, Carlos Banquet Brango, Lucas Ferreira and Élder Villamizar Roa *Universidade Estadual de Campinas, Brazil, Universidad Industrial de Santander, Colombia and Universidad de Córdoba, Colombia cbanquet@correo.unicordoba.edu.co

We study the existence of local and global solutions for coupled Schrödinger-Boussinesq systems with initial data in weak-$L^r$ spaces. These spaces contain singular functions with infinite $L^2$-mass such as homogeneous functions of negative degree. Moreover, we analyze the self-similarity and radial symmetry of solutions by considering initial data with the right homogeneity and radially symmetric, respectively. Since functions in weak-$L^r$ with $r > 2$ have local finite $L^2$-mass, the solutions obtained can be physically realized. Moreover, for initial data in $H^s$, solutions belong to $H^s$ which shows that the constructed data-solution map in weak-$L^r$ recovers $H^s$-regularity.
Consider the Robin and Dirichlet eigenvalue problems

\[
\Delta u + \lambda u = 0 \quad \text{for} \quad x \in \Omega, \\
\frac{\partial u}{\partial \nu} + \alpha u = 0 \quad \text{for} \quad x \in \Gamma, \quad \alpha \in \mathbb{R}, \\
\Delta u + u = 0 \quad \text{for} \quad x \in \Omega, \\
u u = 0 \quad \text{for} \quad x \in \Gamma,
\]

in the bounded domain \( \Omega \subset \mathbb{R}^n, n \geq 2 \) with the boundary \( \Gamma \) of \( C^3 \) class, \( \nu \) is the outward unit normal vector. By \( \{\lambda_k^R(\alpha)\}_{k=1}^{\infty} \) and \( \{\lambda_k^D\}_{k=1}^{\infty} \) we denote the eigenvalues of Robin and Dirichlet problems, enumerated in the increasing order according to their multiplicities. By \( m(\lambda) \) we denote the multiplicity of eigenvalue \( \lambda \).

**Theorem 1.** Let \( m(\lambda_k^D) = 1 \). Then there exists the number \( \alpha_k \in \mathbb{R} \) such that for all \( \alpha > \alpha_k \) we have \( m(\lambda_k^R(\alpha)) = 1 \) and the eigenvalue \( \lambda_k^R(\alpha) \) obeys an asymptotic expansion

\[
\lambda_k^R(\alpha) = \lambda_k^D - \frac{\int_{\Gamma}(\frac{\partial u_k^D}{\partial \nu})^2 \, ds}{\int_{\Omega}(u_k^D)^2 \, dx} \alpha^{-1} + o(\alpha^{-1}), \quad \alpha \to +\infty.
\]

Let \( \{u_k^R(\alpha)\}_{k=1}^{\infty} \) and \( \{u_k^D(x)\}_{k=1}^{\infty} \) are orthogonal and normalized in \( L_2(\Omega) \) sequences of eigenfunctions to Robin and Dirichlet problems. For any \( \alpha \in \mathbb{R} \) we suppose \( \int_{\Omega} u_{k,\alpha}^R u_k^D \, dx \geq 0 \).

**Theorem 2.** Let \( m(\lambda_k^D) = 1 \). Then there exist the constants \( C', C'' > 0 \) such that for \( \alpha > \max\{\alpha_k, 1\} \) the following estimates hold:

\[
\frac{C'}{\alpha} \leq \|u_{k,\alpha}^R - u_k^D\|_{H^2(\Omega)} \leq \frac{C''}{\alpha}.
\]
One of the most important topics in Riemannian geometry is the relation between topology and geometric structure on Riemannian manifolds. The celebrated theorem of S. B. Myers (Duke Math. J. 8 (1941), 401–404) guarantees the compactness of a complete Riemannian manifold under some positive lower bound on the Ricci curvature. This theorem may be considered as a topological obstruction for a complete Riemannian manifold to have a positive lower bound on the Ricci curvature. On the other hand, J. Lohkamp (Ann. of Math. 140 (1994), 655–683) proved that in dimension at least three, any manifold admits a complete Riemannian metric of negative Ricci curvature. Hence, in dimension at least three, there are no topological obstructions to the existence of a complete Riemannian metric of negative Ricci curvature. To give an interesting compactness criterion for complete Riemannian manifolds is one of the most important problems in Riemannian geometry, and the Myers theorem has been widely generalized in various directions by many authors.

The aim of my talk is to discuss the compactness of Ricci solitons. Ricci solitons were introduced by R. Hamilton in 1982 and are natural generalizations of Einstein manifolds. They correspond to self-similar solutions to the Ricci flow and arise as singularity models of the flow. The importance of Ricci solitons was demonstrated by G. Perelman, where Ricci solitons played crucial roles in his affirmative solution of the Poincaré conjecture. In this talk, after we reviewed basic facts on Ricci solitons, we shall establish some new compactness theorems for Ricci solitons. Our results may be regarded as natural generalizations of the compactness theorem due to J. Cheeger, M. Gromov, and M. Taylor (J. Differential Geom. 17 (1982), 15–53) and improve previous compactness theorems for Ricci solitons obtained by M. Fernández-López and E. García-Río, M. Limoncu, Z. Qian, G. Wei and W. Wylie.

**Numerical analysis of a generalized particle method for the convection-diffusion equation**, Daisuke Tagami tagami@imi.kyushu-u.ac.jp

Particle methods such as Smoothed Particle Hydrodynamics (SPH) and Moving Particle Semi-implicit (MPS) are widely used for numerical simulations of flow problems with moving boundaries. However, numerical analysis of particle methods, which can be directly applied into practical computations by SPH and
MPS, is not enough. This is why we focus our attention on numerical analysis of particle methods.

We have introduced generalized particle methods for Poisson and heat equations, and have already established their error estimates. The generalized particle method is a class of particle methods for partial differential equations with strong form, and can describe SPH, MPS, and finite difference methods under appropriate settings. The regularity condition on discrete parameters introduced by us have led to the consistency, stability, and convergency of the generalized particle methods for Poisson and heat equations.

Therefore, we continue our numerical analysis of a generalized particle method for the convection-diffusion equation, where the material derivative appearing in the equation is directly approximated by a semi-implicit characteristic method. This strategy leads two strong points. One strong point is we can apply the regularity condition on discrete parameters in case of Poisson and heat equations. The other one is we can avoid uneven particle distributions at every time steps, which plays key role in numerical stability. The reason of the second point is as follows: First we distribute particles at each time step satisfying the regularity condition without following the motion of particles at each time step. Next, when we are required to refer physical values at the previous time step, we just evaluate interpolants of physical values at the previous time step. Finally, we can estimates interpolant errors by using the previous result.

Moreover, some numerical results are shown, which agree well with theoretical ones.

The Hilbert series and Laurent coefficients of circle invariants, Daniel Herden
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Let $V$ be a finite-dimensional unitary $\mathbb{C}^\times$-module. Choosing a basis for $V$ with respect to which the $\mathbb{C}^\times$-action is diagonal, we can describe the action with a weight vector $\mathbf{a} = (a_1, a_2, \ldots, a_n) \in \mathbb{Z}^n$. Specifically, for $w \in \mathbb{C}^\times$ and $\mathbf{z} \in V$ with coordinates $\mathbf{z} = (z_1, z_2, \ldots, z_n)$, we have

$$w \cdot (z_1, z_2, \ldots, z_n) = (w^{a_1}z_1, w^{a_2}z_2, \ldots, w^{a_n}z_n).$$

Let $\mathbb{C}[V]^{\mathbb{C}^\times}$ denote the associated graded algebra of polynomial $\mathbb{C}^\times$-invariants,
and let \( \text{Hilb}_a(t) \) denote the Hilbert series of \( \mathbb{C}[V]^{\mathbb{C}_a^x} \), the generating function of the dimensions of the homogeneous components of \( \mathbb{C}[V]^{\mathbb{C}_a^x} \).

We will give explicit formulas for \( \text{Hilb}_a(t) \) as well as the first four coefficients of the Laurent expansion of \( \text{Hilb}_a(t) \) at \( t = 1 \). The naive formulas for these coefficients have removable singularities when weights pairwise coincide. Identifying these cancelations, the Laurent coefficients are expressed with the help of partial Schur polynomials that are independently symmetric in two sets of variables. We similarly will present an explicit formula for the \( a \)-invariant of \( \mathbb{C}[V]^{\mathbb{C}_a^x} \) in the case that this algebra is Gorenstein.

**Regressive order on subsets of regular cardinals**, Yinhe Peng, Pedro Sánchez Terraf and William Weiss pengyh@math.utoronto.ca

In this work in progress, we study the partial order relation among subsets of regular cardinals \( \kappa \) given by: \( X <_R Y \) if and only if there exists \( \gamma < \kappa \) and an injective regressive \( f : X \setminus \gamma \to Y \) (i.e., \( f(\alpha) < \alpha \) for all \( \alpha \in X \setminus \gamma \)). Most of our results are about \( \kappa = \omega_1 \).

We show that there are nonstationary subsets of \( \omega_1 \) that are maximal with respect to \( <_R \). We characterize \( <_R \) combinatorially by using integer-valued functions; the main point is the fact that \( X <_R Y \) depends on the relative positions of \( X \) and \( Y \) in their union. Finally, we show that any family of \( \aleph_1 \) subsets of \( \omega_1 \) has a lower bound.

**Multifactorial numbers by context-free grammars**, Juan G. Triana and Rodrigo De Castro jtrianal@unal.edu.co

The formal functions over an alphabet \( \Sigma \) and the derivative operator \( D \), based on substitution rules, were introduced by William Chen in the study of formal power series. Let \( \Sigma \) be an alphabet and \( G \) a context-free grammar over \( \Sigma \), thus the substitution rules on \( G \) are of the form \( a \to u \), where \( a \in \Sigma \) and \( u \) is a formal function. The formal derivative \( D \), with respect to the grammar \( G \), is such that if there is a production \( a \to v \), then \( D(a) = v \); otherwise \( D(a) = 0 \).

Recently, some results have been published connecting context-free grammars
and combinatorial objects such as Eulerian numbers, Stirling numbers, Bessel polynomials, among others. In this presentation we introduce a connection between context-free grammars and multifactorial numbers; some properties about these numbers are proved by grammatical methods.

**Elliptic Problems in Weighted Spaces**, Hovik A. Matevosian hmatevossonian@graduate.org

We study the asymptotic behavior of solutions of elliptic boundary value problems at infinity under the assumption that generalized solutions of these problems have the finite weighted Dirichlet (energy) integral. Moreover, we study the properties of generalized solutions of boundary value problems for the biharmonic Steklov and Steklov–type problems, and also polyharmonic Dirichlet–Navier and Neumann problems in unbounded domains with a compact and non-compact boundaries (in particular, the exterior of a compact set, half-space, the domain with conical points) under the assumption that the Dirichlet (energy) integral with weight $|x|^a$ is finite for such solutions. Admitting different boundary conditions, we used the variation principle and depending on the value of the parameter $a$, we obtained uniqueness (or non-uniqueness) theorems for these solutions or present exact formulas for the dimension of the space of solutions.

**On a maximal subgroup** $2^{4+6}:(A_5 \times 3)$ **of the Chevalley group** $G_2(4)$, Abraham Love Prins Cape Peninsula University of Technology, South Africa

The Chevalley group $G_2(4)$ has a class of maximal subgroups of the form $2^{4+6}:(A_5 \times 3)$. The extension group $G = 2^{4+6}:(A_5 \times 3)$ is isomorphic to two non-split groups of the form $2^6:(2^4:(A_5 \times 3))$ and $2^4:(2^6:(A_5 \times 3))$. In this presentation, the author will construct both the character tables of $G_1$ and $G_2$ by computing two different sets of Fischer matrices for $G_1$ and $G_2$, respectively. Interesting results are obtained in computing the projective character tables of the inertia factor groups for the groups $G_1$ and $G_2$. Having obtain these projective character tables and Fischer matrices, the character tables of the groups $G_1$ and $G_2$ are easily constructed.
Successive Partial Smoothing in the Sparse Plus Low Rank Matrix Decomposition via a Spectral Proximal Gradient Method, Ivan X. Nascimento, Sandra A. Santos and Paulo J. Silva ivan.xmn@gmail.com

This work addresses the fundamental discrete problem of splitting a given $m$-by-$n$ matrix into the sum of its sparse and low-rank components with no further information about sparsity and rank patterns.

As it is common in the literature, sparse and low-rank summands are induced by their classic non-smooth convex surrogates (the componentwise $\ell_1$-norm and the nuclear norm, respectively), which are here convexly combined to form the function to be minimized. Our interest resides in practical examples that come from real applications such as face recognition and video surveillance.

We propose the replacement of either the $\ell_1$-norm term or the nuclear norm term with a smooth function that pointwisely underestimates its value, thus obtaining a convex objective function that is smooth only in one summand. The optimization is then conducted via a proximal gradient method, in which a proximal strategy for the non-smoothed norm is used and the smoothing parameter is adjusted in a continuation scheme.

Illustrative and comparative numerical experiments with data coming from both artificial and real settings are presented. We conclude that as far as the human visual perception is concerned in practice, this approach can even outperform an accelerated version of the Alternating Direction Method of Multipliers in terms of both number of iterations and number of singular value decompositions.

On the valuation of options during high volatility periods-A PDE approach, Youssef A. El-Khatib youssef_elkhatib@uaeu.ac.ae

This paper deals with the pricing problem for European contingent claim for a market suffering from a financial crisis. More precisely, we assume the following dynamics for the underlying asset price $S_t$:

$$dS_t = rS_t dt + (\sigma S + g(t)) dW_t,$$

where $g(t) = A + Be^{\alpha t} \sin(\omega t)$ and $S_0 = x > 0$. Here $(W_t)_{t \in [0,T]}$ is a standard Brownian motion, $\alpha, \beta, \omega, A, B$ and $\sigma$ are all constant. When $g(t) = 0$, the model
is reduced to the Black and Scholes model. If $C(t, S)$ designates the European call option’s price at time $t$, $r$ the risk-free rate, and $K$ the strike price then $C(t, S) \in [0, T] \times ]0, +\infty[$ satisfies the following PDE

$$\frac{\partial C}{\partial t} + rS \frac{\partial C}{\partial S} - rC + \frac{1}{2}(\sigma S_t + g(t))^2 \frac{\partial^2 C}{\partial S^2} = 0,$$

$$C(T, S) = f(S) = (S - K)^+, \quad 0 < S < \infty. \quad (13)$$

We investigate a solution for the European contingent claim pricing PDE (12)-(13).

**Minimal time mean field games**, Guilherme Mazanti guilherme.mazanti@polytechnique.org

Mean field games have been extensively studied since their introduction around 2006 by the independent works of J.-M. Lasry and P.-L. Lions, and P. E. Caines, M. Huang, and R. P. Malhamé. Such differential games with a continuum of indistinguishable agents have been proposed as approximations of games with a large number of symmetric players, finding their applications in several domains.

Motivated by the problem of proposing mean field game models for crowd motion, this talk considers a mean field game where agents want to leave a bounded domain through a part of its boundary in minimal time. Each agent is free to move in any direction, but their maximal speed is assumed to be bounded in terms of the distribution of agents around their position in order to take into account congestion phenomena.

With respect to most mean field game models in the literature, the novelties here are the velocity constraint and the fact that the final time in the optimization criterion is not fixed, which are important features from a modeling point of view for crowd motion, but bring several extra difficulties in the analysis.

After formulating this mean field game in a Lagrangian setting, we establish the existence of Lagrangian equilibria using a fixed point strategy. Thanks to some further regularity properties of optimal trajectories obtained through Pontryagin’s maximum principle, we characterize equilibria through a system of a continuity equation on the distribution of agents coupled with a Hamilton–Jacobi equation.
on the value function of the optimal control problem solved by each agent. We also provide sufficient conditions ensuring the $L^p$ regularity of the distribution of agents, and conclude the talk by discussing further extensions of the model and some open questions.

This talk is based on ongoing joint works with Samer Dweik and Filippo Santambrogio.

**Scaled consensus of switched multi-agent systems**, Yilun Shang  
shylmath@hotmail.com

In the past decades, distributed control of networked multi-agent systems has received a great deal of attention in the system and control community due to its broad applications in areas such as formation control, cooperative coordination of unmanned aerial vehicles, robotic teams and sensor networks. In this talk, we discuss the scaled consensus problem for switched multi-agent systems composed of continuous-time and discrete-time subsystems. By using the nearest neighbor-interaction rules, three types of scaled consensus protocols are presented to achieve asymptotic, finite-time, and fixed-time convergence rates, respectively. Based upon algebraic graph theory and Lyapunov theory, scaled consensus is shown to be reached for strongly connected networks. We explicitly express the final consensus states as some initial-condition-dependent values. The asymptotic, finite-time and fixed-time scaled formation control problems for switched multi-agent systems are introduced and solved as the generalizations. Simulation examples are provided to illustrate the effectiveness and availability of our theoretical results.

N.B. The paper has been accepted by IMA Journal of Mathematical Control and Information.

**Analysis of mixed interfacial crack problems for the system of dynamics of Thermo-electro-magneto-elasticity theory**, David Natroshvili  
natrosh@hotmail.com

We investigate regularity properties of solutions to mixed boundary value
problems for the system of partial differential equations of dynamics associated with the thermo-electro-magneto elasticity theory. We consider piecewise homogeneous anisotropic elastic solid structures with interior and interfacial cracks, in particular, smart materials and structures. Using the potential method and theory of pseudodifferential equations we prove the existence and uniqueness of solutions. The singularities and asymptotic behaviour of the thermo-mechanical and electro-magnetic fields are analyzed near the crack edges and near the curves, where different types of boundary conditions collide. In particular, for some important classes of anisotropic media we derive explicit expressions for the corresponding stress singularity exponents and demonstrate their dependence on the material parameters. The questions related to the so called oscillating singularities are analyzed in detail as well.

Statistical inference for sparse binary chains of unbounded memory, Florencia Leonardi florencia@usp.br

In this work we consider the problem of estimation of a sparse context tree for a class of stochastic processes with binary alphabet. The context tree of a stochastic process is a structure that summarizes the relevant past sequences that are necessary to predict the present symbol of the process. By a sparse context tree we mean a classical context tree with some “junction of nodes” at specific positions in the past sequences; that is assuming the relevant contexts in the tree are “sparse” and have many positions that are not relevant to predict the present state of the stochastic chain. We propose to use a penalized criterion to select the sparse contexts of the process, and take advantage of the relationship between maximum likelihood estimation for binary processes with logistic regression problems. We regularize the criterion with an $\ell_1$ norm on the regression parameters and show how to efficiently compute the context tree estimator using a LASSO type estimator. We show that this criterion is consistent in the sense that the truncated sparse context tree can be recovered with probability converging to 1 when the sample size increases, allowing the maximal depth of the tree to grow simultaneously as a function of the sample size.
Optimal Consumption, investment and life-insurance purchase under a stochastically fluctuating economy, Abdelrahim Mousa, Diogo Pinheiro and Alberto Pinto asaid@birzeit.edu

We study the optimal consumption, investment and life-insurance purchase and selection strategies for a wage-earner with an uncertain lifetime with access to a financial market comprised of one risk-free security and one risky-asset whose prices evolve according to linear diffusions modulated by a continuous-time stochastic process determined by an additional diffusive nonlinear stochastic differential equation. The process modulating the linear diffusions may be regarded as an indicator describing the state of the economy in a given instant of time. Additionally, we allow the Brownian motions driving each of these equations to be correlated. The life-insurance market under consideration herein consists of a fixed number of providers offering pairwise distinct contracts. We use dynamic programming techniques to characterize the solutions to the problem described above for a general family of utility functions, studying the case of discounted constant relative risk aversion utilities with more detail.

Adaptive Simulation of Rarefied Gases Using Hierarchical Moment Models, Julian Koellermeier koellermeier@zedat.fu-berlin.de

The numerical simulation of high-altitude flows often involves regions with dense as well as rarefied gases. In dense regions standard fluid dynamics equations like the Euler equations can be readily used to efficiently compute the flow field. However, in order to resolve non-equilibrium effects in rarefied gases accurate mathematical models require additional variables and thus lead to more runtime and memory demand in simulations. It thus poses a challenging task and requires special coupled mathematical models to balance the necessary accuracy in the rarefied domain with the computational efficiency in the dense part of the gas.

A new adaptive simulation framework for the numerical simulation involving rarefied gases will be presented in this talk. Instead of solving the complex rarefied model in the whole domain, adaptive hierarchical domain decomposition is performed and the model is adjusted to match accuracy requirements in each respective domain.
The main feature is the underlying hierarchical moment model for rarefied gases that allows adjusting the necessary accuracy depending on the degree of rarefaction of the gas. The moment model yields a non-linear, hyperbolic PDE in partially conservative form that is solved using path-conservative numerical schemes. In dense regions, the model automatically reduces to the Euler equations. Several techniques for error detection and refinement as well as coarsening of the model will be explained and compared in application cases. It will be shown that both simulation runtime and memory consumption can be reduced while error bounds are satisfied due to the new adaptive simulation framework.

**Global bifurcation analysis of multi-parameter polynomial dynamical systems**, Valery Gaiko  
valery.gaiko@gmail.com

We study multi-parameter polynomial dynamical systems and carry out the global bifurcation analysis of such systems. To control the global bifurcations of limit cycle in planar systems, it is necessary to know the properties and combine the effects of all their rotation parameters. It can be done by means of the development of a new bifurcational geometric method based on the Wintner–Perko termination principle stating that the maximal one-parameter family of multiple limit cycles terminates either at a singular point which is typically of the same multiplicity (cyclicity) or on a separatrix cycle which is also typically of the same multiplicity (cyclicity). If we do not know the cyclicity of the termination points, then, applying canonical systems with field rotation parameters, we use geometric properties of the spirals filling the interior and exterior domains of limit cycles. Using this method, we solve, e.g., Hilbert’s Sixteenth Problem on the maximum number of limit cycles and their distribution for the general Liénard polynomial system, Holling-type quartic dynamical system, and Kukles cubic-linear system. Applying a similar approach, we study also three-dimensional polynomial dynamical systems and, in particular, complete the strange attractor bifurcation scenario in the classical Lorenz system connecting globally the homoclinic, period-doubling, Andronov–Shilnikov, and period-halving bifurcations of its limit cycles.

**On Some Applications Of Measures Of Noncompactness**, M. Mursaleen
In this talk, we present a brief survey of theory and applications of measures of noncompactness. The classical measures of noncompactness are discussed and their properties are compared. The approaches for constructing measure of noncompactness in a general metric or linear space are described, along with the classical results for existence of fixed point for condensing operators. Also several generalization of classical results are mentioned and their applications in various problems of analysis such as linear equation, differential equations, integral equations and common solutions of equations are discussed.

The most effective way in the characterization of compact operators between the Banach spaces is applying the Hausdor measure of noncompactness. In this chapter, we present some identities or estimates for the operator norms and the Hausdor measures of noncompactness of certain operators given by infinite matrices that map an arbitrary $BK$-space into the sequence spaces $c_0, c, \ell_\infty$ and $\ell_1$. Many linear compact operators may be represented as matrix operators in sequence spaces or integral operators in function spaces, see J. Banas and M. Mursaleen, Sequence Spaces and Measures of Noncompactness with Applications to Differential and Integral Equations, Springer, 2014.

We apply the technique of measures of noncompactness to the theory of infinite systems of differential equations in some Banach sequence spaces $c_0, c, \ell_p$ ($1 \leq p < \infty$). Infinite systems of ordinary differential equations describe numerous world real problems which can be encountered in the theory of branching processes, the theory of neural nets, the theory of dissociation of polymers and so on. Let us also mention that several problems investigated in mechanics lead to infinite systems of differential equations. Moreover, infinite systems of differential equations can be also used in solving some problems for parabolic differential equations investigated via semidiscretization. We adopt the technique of measures of noncompactness to the theory of infinite systems of differential equations. Particularly, we are going to present a few existence results for infinite systems of differential equations formulated with the help of convenient and handy conditions. We study of the solvability of the infinite systems of differential equations in some classical Banach sequence spaces.
Truncated long-range percolation on oriented graphs, Bernardo N. de Lima
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We consider different problems within the general theme of long-range percolation on oriented graphs. Our aim is to settle the so-called truncation question, described as follows. We are given probabilities that certain long-range oriented bonds are open; assuming that the sum of these probabilities is infinite, we ask if the probability of percolation is positive when we truncate the graph, disallowing bonds of range above a possibly large but finite threshold. We give some conditions in which the answer is affirmative. We also translate some of our results on oriented percolation to the context of a long-range contact process. Based on joint works with Caio Alves, Aernout van Enter, Marcelo Hilário and Daniel Valesin.

Well-posedness and asymptotic stability for a Bresse system with infinite memories, Aissa Guesmia
Elie Cartan Institute, University of Lorraine, Metz, France aissa.guesmia@univ-lorraine.fr

In this work, we consider a linear Bresse system in one-dimensional bounded interval with at least one infinite memory acting in at least one of the three equations of the system (vertical, shear angle and longitudinal displacements). First, using the semigroup theory, we prove that the Bresse system is well posed. Second, we prove two general and precise (uniform and weak) stability estimates depending on the speeds of wave propagations, the number of considered memories and the growth of the convolution kernels at infinity, where the convolution kernels can have a decay rate arbitrary close to $\frac{1}{s}$. In case of weak stability, the decay rate depends also on the smoothness of initial data.

The case of three memories is a joint work with Mohammad Kafini (KFUPM, Dhahran, Saudi Arabia), where the results have been published in Math. Meth. Appl. Scie., 2014.

The case of two memories is a joint work with Mokhtar Kirane (Université de la Rochelle, France), where the results have been published in ZAMP, 2016.

The case of one memory acting on the shear angle displacement is a joint
work with Salim Messaoudi (KFUPM, Dhahran, Saudi Arabia), where the paper is submitted.

The case of one memory acting on the longitudinal displacement has been published in Med. J. Math., 2017.

The case of one memory acting on the vertical displacement has been published in Nonaton. Dyn. Syst., 2017.

**Lefschetz properties for Artinian Gorenstein algebras presented by quadrics**, Rodrigo Gondim and Giuseppe Zappalà rodrigo.gondim@ufrpe.br

We introduce a family of Artinian Gorenstein algebras, whose combinatorial structure characterizes the ones presented by quadrics. Under certain hypothesis these algebras have non-unimodal Hilbert vector. In particular we provide families of counterexamples to the Migliori-Nagel Conjecture that Artinian Gorenstein algebras presented by quadrics should satisfy the weak Lefschetz property.

**Ruelle Zeta Function at Zero for Hyperbolic Surfaces with Finite Volume**, Lee Peng Teo lpteo@xmu.edu.my

In this work, we consider a general hyperbolic surface $X = \Gamma \backslash \mathbb{H}$ of genus $g$ with $n$ punctures and $r$ ramification points of ramification indices $m_1, \ldots, m_r$. The Ruelle zeta function of such a surface is defined as

$$
\zeta_R(s) = \prod_P \left(1 - e^{-s \ell_P}\right)
$$

where $P$ runs through the set of primitive closed geodesics on $X$ and $\ell_P$ is the length of $P$. In this work, we consider the behavior of the Ruelle zeta function at $s = 0$. We prove that as $s \to 0$,

$$
\zeta_R(s) = (-1)^{\frac{g}{2} + 1}(2\pi s)^{2g-2+n} \prod_{j=1}^{r} m_j + \text{higher order terms},
$$
where $A$ is an even integer given by

$$A = n - \text{Tr} \Phi \left( \frac{1}{2} \right),$$

with $\Phi$ the scattering matrix. This is the first such result for surfaces with conical singularities. We use Selberg trace formula to obtain the result.

**Statistical Evaluation of DWT-PCA/SVD and FFT-PCA/SVD Face Recognition Algorithms under varying Head-poses**, Louis Asiedu, Atinuke O. Adebani and Favour N. Yirenkyi lasiedu@ug.edu.gh

Automated intelligent systems have been developed to imitate the ability to recognize faces inherent in human beings. An efficient and resilient recognition system is useful in many application areas such as, surveillance, security systems based on biometric data and missing children identification.

Despite these successes of automated systems, face recognition algorithms are still faced with the challenge of recognizing faces under varying constraints (illumination, facial expressions and head-poses). That is, although remarkably robust, face recognition systems are not perfectly invariant to pose and viewpoint changes. This study statistically evaluated the performance of Principal Component Analysis with Singular Value Decomposition using Discrete Wavelet Transform for pre-processing (DWT-PCA/SVD) and Principal Component Analysis with Singular Value Decomposition using Fast Fourier Transform for pre-processing (FFT-PCA/SVD) face recognition algorithms under specified head-poses ($4^\circ, 8^\circ, 12^\circ, 16^\circ, 20^\circ, 24^\circ, 28^\circ$ and $32^\circ$).

Ten face images from 10 subjects captured under straight-pose ($0^\circ$) were used for training in the face recognition module. Eighty face images from 10 subjects captured under the specified head-poses were used for testing. Multivariate statistical methods (repeated measures, paired comparison, Box’s M test, profile Analysis) were used to evaluate the algorithms under varying head-poses.

The results of the study showed that, FFT-PCA/SVD has a higher average recognition rate (92.5%) and lower variation in recognition distance making it more efficient and consistent under varying head-poses. The study also found that FFT-PCA/SVD recognizes perfectly (100% recognition rate) head-poses that are
24° and below. FFT-PCA/SVD is therefore recommended for recognition of face images under varying head-poses.

**Anomalous diffusion in a system of harmonic oscillators perturbed by a conservative noise**, Milton Jara mjara@impa.br

We review the derivation of the fractional heat equation with self-similarity exponent $3/2$ as the scaling limit of energy fluctuations on an harmonic chain of oscillators with conservative noise. This derivation provides a rigorous proof of the anomalous diffusive behaviour of heat conduction models in one dimension, and it also clarifies the role of the conservation of momentum in this anomalous behaviour.

**On Rigid 2-Step Nilpotent Lie Algebras**, María A. Alvarez maria.alvarez@uantof.cl

In this note we consider 2-step nilpotent Lie algebras and give a criterion for the rigidity of this class in the variety $N^2_n$ of 2-step nilpotent Lie algebras of dimension $n$. We apply this criterion to prove that every rigid Lie algebra in $N^2_n$ is indecomposable, except for $\mathfrak{h}_3 \oplus \mathbb{C}$ and $\mathfrak{h}_3 \oplus \mathfrak{h}_3$.

**Variational convergence of vector functions and its application to multiobjective optimization**, Rubén L. López rlopezm@academicos.uta.cl

We recall a notion of variational convergence for vector functions that is suitable for studying multiobjective optimization problems. We obtain properties of the variational convergence and characterize it via the set convergence of epigraphs, coepigraphs, level sets, and some infima. We provide a metric characterization of this convergence and we compare it with other convergence notions from the literature. Finally, we employ this convergence notion to obtain global stability and well-posedness results for multiobjective optimization problems.
Classical approximation theory and its application in functional analysis, Thaís Jordão tjordao@icmc.usp.br

K-functionals, moduli of smoothness and rate of approximation of average operators play an important role in classical approximation theory since they express intrinsic properties related to the smoothness of a function.

Since 2013, in a new technique have been used in order to improve results in functional analysis concerning about to get sharp estimates for the eigenvalue sequences of certain integral operators on spaces of complex valuable functions on the unit sphere. Researches on estimates for the eigenvalue sequences as mentioned here has the last big contribution at 80’s. Since there it has been staked until 2013, that is why this technique figures as an important contribution in this research area.

This short talk is about of new developments on approximation theory and its consequences in the study of eigenvalues sequences of integral operators on a general setting, namely homogeneous spaces of rank 1.

Recent advances on the branch-and-prune algorithm for the molecular distance geometry problem, Michael Souza, Carlile Lavor and Luiz M. Carvalho michael@ufc.br

The molecular distance geometry problem (MDGP) is associated to protein structure calculations using Nuclear Magnetic Resonance (NMR) data. The problem is how to calculate the atomic positions using NMR distance information. NMR data and protein geometry allow us to define atomic orders, such that a combinatorial method, called Branch-and-Prune (BP), can be applied to the problem. Based on new orders and new versions of BP algorithm, we present the most recent results and new research directions related to the application of BP approach to the MDGP.

Gerstenhaber brackets in Hochschild cohomology, Andrea Solotar asolotar@dm.uba.ar
Hochschild cohomology and its Gerstenhaber algebra structure are relevant invariants: they are invariant by Morita equivalences, by tilting processes and by derived equivalences. The computation of these invariants requires a resolution of the algebra considered as a bimodule over itself. Of course, there is always a canonical resolution available, the bar resolution, very useful from a theoretical point of view, but not very satisfactory in practice: the complexity of this resolution rarely allows explicit calculations to be carried out.

Recently, some important advances have been obtained in this direction. Here, we develop strategies well adapted to different types of algebras: the Jordan and the super Jordan plane –both of them are Nichols algebras–, a family of special biserial algebras, and some subalgebras of the Weyl algebra. These strategies allow the complete computation of Gerstenhaber brackets, the description of the first cohomology space as a Lie algebra and the Lie module structure of the higher cohomology spaces.

The talk is based on results obtained in a joint work with Sebastián Reca, and work in progress with Samuel Lopes and with Joanna Meinel, Van Nguyen, Bregje Pauwels and María Julia Redondo.

A high-order conservative finite element formulation for Darcy flow problem, Eduardo Abreu, Ciro Ciro Diaz, Juan Galvis and Marcus Sarkis eabreu@ime.unicamp.br

A novel high-order conservative finite element method for Darcy flow is introduced and analyzed. The new ingredient in the formulation is a residual-based volumetric constrain, based on Lagrange multipliers in order to impose locally mass conservation that does not involve any mesh grid dependent parameters. We establish high-order a priori error estimates with locally conservative fluxes and numerics are presented and discussed to confirm the theoretical analysis. Our approach can be straightforwardly extended to three dimensions and it is also applicable to highly heterogeneous and anisotropic problems, where high order approximations is necessary for accurate Darcy velocity fields linked to porous media flow and transport problems.
Cosmetic surgeries on knots, Kazuhiro Ichihara, Tetsuya Ito, In Dae Jong, Toshio Saito and Zhongtao Wu ichihara.kazuhiro@nihon-u.ac.jp

Dehn surgery on a knot along a slope can be regarded as an operation to make a “new” 3-manifold from a given one. Of course the trivial Dehn surgery leaves the manifold unchanged, but “most” non-trivial ones would change the topological type. In view of this, a pair of Dehn surgeries on a knot along slopes are called purely cosmetic (resp. chirally cosmetic) if there exists an orientation preserving (resp. orientation reversing) homeomorphism between the pair of the surgered manifolds. In this talk, I will report several recent results on purely and chirally cosmetic surgeries.

A Unified System of FB-SDEs with Levy Jumps and Double Completely-Skew Reflections, Wanyang Dai nan5lu8@nju.edu.cn

We study the well-posedness of a unified system of coupled forward-backward stochastic differential equations (FB-SDEs) with Lévy jumps and double completely-S skew reflections. Owing to the reflections, the solution to an embedded Skorohod problem may be not unique, i.e., bifurcations may occur at reflection boundaries, the well-known contraction mapping approach cannot be extended directly to solve our problem. Thus, we develop a weak convergence method to prove the well-posedness of an adapted 6-tuple weak solution in the sense of distribution to the unified system. The proof heavily depends on newly established Malliavin calculus for vector-valued Lévy processes together with a generalized linear growth and Lipschitz condition that guarantees the well-posedness of the unified system even under a random environment. Nevertheless, if a more strict boundary condition is imposed, i.e., the spectral radii in certain sense for the reflections are strictly less than the unity, a unique adapted 6-tuple strong solution in the sense of sample pathwise is concerned. In addition, as applications and economical studies of our unified system, we also develop new techniques including deriving a generalized mutual information formula for signal processing over possible non-Gaussian channels with multi-input multi-output (MIMO) antennas and dynamics driven by Lévy processes.
Blocks of finite groups, fusion systems and cohomology, Constantin-Cosmin Todea constantin.todea@math.utcluj.ro

Let \( k \) be a field of characteristic \( p \). A block of a finite group \( G \) is an idempotent of the center of the group algebra \( kG \) which is not decomposable as sums of other orthogonal idempotents from the same center. By the work of Alperin, Puig and Broué in the late 70’s we associate to a block a \( G \)-poset of Brauer pairs. These Brauer pairs can be organized to give a first example of categories called fusion systems. A saturated fusion system on a finite \( p \)-group \( S \) is a category with objects the subgroups of \( S \) and whose morphisms mimics the behavior of conjugacy group homomorphisms between the subgroups in a Sylow \( p \)-subgroup of a given finite group. We present an extension of a theorem of Mislin from groups to fusion systems (when \( p \) is odd), which shows that an equality of mod-\( p \) cohomology algebras of some fusion systems implies the equality of the fusion systems. Some applications of this result with respect to blocks may also be presented.

We also present some aspects of a generalization of blocks: the primitive idempotents of the subalgebra \( (kH)^G \) in \( kH \) of \( G \)-stable elements; where \( H \) is a normal subgroup of a finite group \( G \) which acts by conjugation on \( H \). Using a generalization of ordinary block induction to the case of a primitive idempotent in \( (kH)^G \) (denoted by \( b \)), we show that, when the hyperfocal subgroup is abelian, the fusion system associated to \( b \), respectively to the induced stable primitive idempotent of the generalized Brauer pair corresponding to the hyperfocal subgroup, are the same.

Some results regarding: Hochschild cohomology, block cohomology and group-graded basic Morita equivalences may be presented. This communication is based on two articles, written as unique author, published in 2015 and 2018.

Global bifurcation analysis of multi-parameter polynomial dynamical systems, Valery Gaiko valery.gaiko@gmail.com

We study multi-parameter polynomial dynamical systems and carry out the global bifurcation analysis of such systems. To control the global bifurcations of limit cycle in planar systems, it is necessary to know the properties and combine the effects of all their rotation parameters. It can be done by means of the de-
Development of a new bifurcational geometric method based on the Wintner–Perko termination principle stating that the maximal one-parameter family of multiple limit cycles terminates either at a singular point which is typically of the same multiplicity (cyclicity) or on a separatrix cycle which is also typically of the same multiplicity (cyclicity). If we do not know the cyclicity of the termination points, then, applying canonical systems with field rotation parameters, we use geometric properties of the spirals filling the interior and exterior domains of limit cycles. Using this method, we solve, e. g., Hilbert’s Sixteenth Problem on the maximum number of limit cycles and their distribution for the general Liénard polynomial system, Holling-type quartic dynamical system, and Kukles cubic-linear system. Applying a similar approach, we study also three-dimensional polynomial dynamical systems and, in particular, complete the strange attractor bifurcation scenario in the classical Lorenz system connecting globally the homoclinic, period-doubling, Andronov–Shilnikov, and period-halving bifurcations of its limit cycles.

Unified Integrals involving the product of General Classes of Polynomials of Several Variables, Harish Nagar and Ajay K. Tripathi Sangam University, Bhilwara, Rajasthan and Mewar University, Gangrar, Chittorgarh, Rajasthan (India) drharishngr@gmail.com

This Paper presents certain unified integrals involving the product of general classes of polynomials with the H-function of several variables. This paper deals with integrals associated with general classes of polynomials of one and several variables, Laguerre function of arbitrary order and the H-function of several variables.

As applications of the integrals established here, some known results are exhibited. Certain results involving the Sinha polynomials, Humbert polynomial, Gegenbauer polynomials and the product of several Laguerre polynomials and Jacobi polynomials with the multivariable H-function are derived here.

Special Lagrangian and deformed Hermitian Yang-Mills on tropical manifold, Hikaru Yamamoto hyamamoto@rs.tus.ac.jp
From string theory, the notion of deformed Hermitian Yang–Mills connections has been introduced by Mariño, Minasian, Moore and Strominger. After that, Leung, Yau and Zaslow proved that it naturally appears as mirror objects of special Lagrangian submanifolds via Fourier–Mukai transform between dual torus fibrations. In their paper, some conditions are imposed for simplicity. In this session, I would like to propose data to glue their construction on tropical manifolds and generalize the correspondence without the assumption that the Lagrangian submanifold is a section of the torus fibration.

Magnetic trajectories on the unit tangent bundle of a Riemannian manifold, Marian Ioan Munteanu marian.ioan.munteanu@gmail.com

Magnetic curves represent the trajectories of the charged particles moving on a Riemannian manifold under the action of the magnetic fields. They are modeled by a second order differential equation, that is $\nabla_{\gamma'} \gamma' = \phi \gamma'$, usually known as the Lorentz equation. Such curves are sometimes called also magnetic geodesics since the Lorentz equation generalizes the equation of geodesics under arc-length parametrization, namely, $\nabla_{\gamma} \gamma' = 0$. In the last years, magnetic curves were studied in Kähler manifolds and Sasakian manifolds, respectively, since their fundamental 2-forms provide natural examples of magnetic fields.

In this talk we present our recent investigation on magnetic curves on the unit tangent bundle of a Riemannian manifold $M$. We write the equation of motion for arbitrary $M$. In the case when $M$ is a space form $M(c)$, we prove that every contact magnetic curve is slant. If $c \neq 1$, a contact normal magnetic curve is slant if and only if it satisfies a conservation law. These results generalize the beautiful paper of Klingenberg and Sasaki published in 1975 about geodesics on the unit tangent bundle of the 2-sphere.

The most part of the results presented in this talk are obtained in collaboration with Professor Jun-ichi Inoguchi, University of Tsukuba, Japan.

On the distance spectrum of distance hereditary graphs, Anu Varghese Varghese and Ambat Vijayakumar anukarintholil@gmail.com
Distance-hereditary graphs are connected graphs in which all induced paths are isometric. Let $G$ be a graph with induced subgraph $H$. If we consider the adjacency matrices of $G$ and $H$, the eigenvalues of $G$ and $H$ interlace by Cauchy’s interlacing theorem. This is not true in general in the case of distance matrices. We found a new characterization of distance-hereditary graphs using distance spectrum. We proved that, a graph is distance-hereditary iff for every induced subgraph $H$ of $G$ and $K$ of $H$, the distance eigenvalues of the pairs $(G, H)$ and $(H, K)$ are interlacing, respectively. Also, we compare graph parameters like distance spectral radius and distance energy of $G$ to the parameters of its induced subgraphs. We have analysed the distance spectrum of cographs and found that the interval $(-2,-1)$ is distance eigenvalue free. Also, we obtained the multiplicity of $-2$ and $-1$ in the distance spectrum of cographs.

**Relations between the classical Zagreb indices of graphs**, Batmend Horoldagva

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For a graph $G = (V, E)$, the classical first Zagreb index $M_1$ and the second Zagreb index $M_2$ are defined as: $M_1(G) = \sum_{u \in V(G)} (d_G(u))^2$ and $M_2(G) = \sum_{uv \in E(G)} d_G(u)d_G(v)$, where $d_G(u)$ is the degree of vertex $u$ in $G$. The difference of the classical Zagreb indices is closely related to the vertex-degree-based graph invariant $RM_2$, which is called the reduced second Zagreb index. In 2007, it was conjectured that for each simple graph $G$ with $n$ vertices and $m$ edges, the inequality $M_2(G)/m \geq M_1(G)/n$ holds. Although this conjecture does not hold in general, it was the beginning of a long series of studies in which the validity or non-validity of this inequality for various classes of graphs. In this talk, we concentrate on the above inequality and difference of the classical Zagreb indices. Moreover, the reduced second Zagreb index and its generalization are discussed for some special classes of graphs.

**A new analysis of non-smooth convex optimization problems by using non-Newtonian calculus**, Ali H. Tor  
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In this study, *-directional derivative and *-subgradient are defined using the
multiplicative derivative, making a new contribution to non-Newtonian calculus for use in nonsmooth analysis. As for directional derivative and subgradient, which are used in the nonsmooth optimisation theory, basic definitions and preliminary facts related to optimization theory are stated and proved. Then, the *-subgradient concept is illustrated by providing some examples, such as absolute value and exponential functions. In addition, necessary and sufficient optimality conditions are obtained for convex problems.

**Difference of Sum-of-Squares Convex Programming Formulations for Polynomial Optimization**, Yi-Shuai Niu  

Polynomial optimization is a special case of DC (Difference-of-Convex functions) programming, however representing a multivariate polynomial into a DC function is a difficult task. We propose in this paper some new results on DC programming formulations for polynomial optimization. We will focus on polynomial decomposition techniques. Firstly, we propose four algorithms to formulate any multivariate polynomial into DSOS (Difference-of-Sums-Of-Squares), and then extend these approaches to generate DCSOS (Difference-of-Convex-Sums-Of-Squares) decompositions (i.e., DC decompositions). We prove that DSOS and DCSOS decompositions for any polynomial can be constructed in polynomial time. Some numerical results of our proposed decomposition methods are reported.

**Optimal Consumption, investment and life-insurance purchase under a stochastically fluctuating economy**, Abdelrahim Mousa

We study the optimal consumption, investment and life-insurance purchase and selection strategies for a wage-earner with an uncertain lifetime with access to a financial market comprised of one risk-free security and one risky-asset whose prices evolve according to linear diffusions modulated by a continuous-time stochastic process determined by an additional diffusive nonlinear stochastic differential equation. The process modulating the linear diffusions may be regarded as an indicator describing the state of the economy in a given instant of
time. Additionally, we allow the Brownian motions driving each of these equations to be correlated. The life-insurance market under consideration herein consists of a fixed number of providers offering pairwise distinct contracts. We use dynamic programming techniques to characterize the solutions to the problem described above for a general family of utility functions, studying the case of discounted constant relative risk aversion utilities with more detail.

**Generalized fractional Kinetic equations associated with Aleph functions**, Dinesh Kumar and Junesang Choi  
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Fractional kinetic equations are investigated in order to describe various phenomena governed by anomalous reaction in dynamical systems with chaotic motion. Due to a great importance of the kinetic equations in certain astrophysical problems, their solutions have been investigated by a number of researchers. Recently, the authors used the Sumudu transform to present a generalized solution of the fractional kinetic equations expressed in terms of Aleph functions. Here, by mainly using the Laplace transform, the authors aim at deriving a solution of the fractional kinetic equation which is also expressed in terms of Aleph functions. The main solution presented here is general enough to be specialized to include many earlier results associated with the kinetic equations.

**Scaled consensus of switched multi-agent systems**, Yilun Shang  
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In the past decades, distributed control of networked multi-agent systems has received a great deal of attention in the system and control community due to its broad applications in areas such as formation control, cooperative coordination of unmanned aerial vehicles, robotic teams and sensor networks. In this talk, we discuss the scaled consensus problem for switched multi-agent systems composed of continuous-time and discrete-time subsystems. By using the nearest neighbor-interaction rules, three types of scaled consensus protocols are presented to achieve asymptotic, finite-time, and fixed-time convergence rates, respectively. Based upon algebraic graph theory and Lyapunov theory, scaled consensus is
shown to be reached for strongly connected networks. We explicitly express the final consensus states as some initial-condition-dependent values. The asymptotic, finite-time and fixed-time scaled formation control problems for switched multi-agent systems are introduced and solved as the generalizations. Simulation examples are provided to illustrate the effectiveness and availability of our theoretical results.

N.B. The paper has been accepted by IMA Journal of Mathematical Control and Information.

**Semivectorial bilevel optimization on Riemannian manifolds**, Henri Bonnel, Léonard Todjihoundé and Constantin Udriste [henri.bonnel@univ-nc.nc](mailto:henri.bonnel@univ-nc.nc)

We deal with the Semivectorial bilevel problem in the Riemannian setting. The upper level is a scalar optimization problem to be solved by the leader, and the lower level is a multiobjective optimization problem to be solved by several followers acting in a cooperative way inside the greatest coalition and choosing among Pareto solutions with respect to a given ordering cone.

For the so-called optimistic problem, when the followers choice among their best responses is the most favorable for the leader, we give optimality conditions. Also for the so-called pessimistic problem, when there is no cooperation between the leader and the followers, and the followers choice may be the worst for the leader, we present an existence result.

**An implosion arising from saddle connection in 2D complex dynamics**, Shizuo Nakane [nakane@gen.t-kougei.ac.jp](mailto:nakane@gen.t-kougei.ac.jp)

We consider regular polynomial skew products $f$ on $\mathbb{C}^2$. It is of the form $f(z, w) = (p(z), q_z(w))$, where $p(z)$ and $q(z, w) := q_z(w)$ are polynomials of degree $d \geq 2$. Then the $k$-th iterate $f^k$ of $f$ is written by

$$f^k(z, w) = (p^k(z), Q_z^k(w)) := (p^k(z), q_{p^{k-1}(z)} \circ \cdots \circ q_z(w)).$$

We consider fiber Julia sets $J_z$ for $z \in K_p$, the filled Julia set of $p$. The set $J_z$ is characterized by the set of points $w \in \mathbb{C}$ such that the family $\{f^k(z, \cdot) : \mathbb{C} \rightarrow \}$

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P^2; \ k \geq 0\} \text{ is not normal in any neighborhood of } w.

The map \( f \) is said to have a saddle connection between two saddle fixed points if the unstable set of one saddle intersects the stable set of another saddle.

Assume that \( p \) has an attracting fixed point at \( 0 \), whose immediate basin \( U \) has a repelling fixed point \( \beta \) on its boundary. We also assume that \( q_z(0) \equiv 0 \), \( |q'_0(0)| > 1 \) and \( |q'_\beta(0)| < 1 \). Then \((0, 0)\) and \((\beta, 0)\) are saddle fixed points and have a saddle connection. Numerical pictures show that \( J_z \) oscillates and behave discontinuously as \( z \to \beta \). This phenomenon quite resembles the parabolic implosion, and we explain such behaviors of fiber Julia sets by an analogous argument as parabolic implosion. A key tool in parabolic implosion is the Fatou coordinates. Instead, we use the linearizing coordinates at both saddle fixed points.

Take a sequence \( z_n \to \beta \) in \( U \). Under some assumptions, we can show that there exists a sequence \( k_n \to \infty \) such that the fiberwise high iterates \( Q_{z_n}^{k_n} \) converge to a map \( g \). The map \( g : W^s((\beta, 0)) \to W^u((0, 0)) \) is the Lavaurs map. Then \( J_{z_n} \) converges to the fiber Julia-Lavaurs set, which is defined analogously as in parabolic implosion. This is a joint work with H. Inou.

**Small-time global stabilization of the viscous Burgers equation with three scalar controls**, Jean-Michel Coron and Shengquan Xiang coron@ann.jussieu.fr

We consider the following viscous Burgers controlled system:

\[
\begin{align*}
  y_t - y_{xx} + yy_x &= \alpha(t) \quad \text{for } (t, x) \in (s, +\infty) \times (0, 1), \\
  y(t, 0) &= u_1(t) \quad \text{for } t \in (s, +\infty), \\
  y(t, 1) &= u_2(t) \quad \text{for } t \in (s, +\infty), \\
  a_t &= \alpha(t) \quad \text{for } t \in (s, +\infty),
\end{align*}
\]

where, at time \( t \), the state is \((y(t, \cdot), a(t)) \in L^2(0, 1) \times \mathbb{R}\), and the control is \((\alpha(t), u_1(t), u_2(t)) \in \mathbb{R} \times \mathbb{R} \times \mathbb{R}\).

We construct explicit time-varying feedback laws leading to the global (null) stabilization in small time of the viscous Burgers equation with three scalar controls. Our feedback laws use first the quadratic transport term to achieve the small-time global approximate stabilization and then the linear viscous term to get the
small-time local stabilization. For closed-loop system with the feedback law, let us define the flow $\Phi: \Delta \times (L^2(0,1) \times \mathbb{R}) \to (L^2(0,1) \times \mathbb{R})$, with $\Delta := \{(t,s) \mid t > s\}$ associated to this feedback law: $\Phi(t,s; y_0, a_0)$ is the value at $t > s$ of the solution $(y,a)$ to the system which is equal to $(y_0, a_0)$ at time $s$. We have the following theorem:

**Theorem** Let $T > 0$. There exists a proper $2T$-periodic time-varying feedback law for the preceding system such that

(i) $\Phi(4T + t, t; y_0, a_0) = 0, \ \forall t \in \mathbb{R}, \ \forall y_0 \in L^2(0,1), \ \forall a \in \mathbb{R}$.

(ii) (Uniform stability property.) For every $\delta > 0$, there exists $\eta > 0$ such that

$$\| (y_0, a_0) \|_V \leq \eta \Rightarrow (\| \Phi(t, t'; y_0, a_0) \|_V \leq \delta, \ \forall t' \in \mathbb{R}, \ \forall t \in (t', +\infty)).$$

**Chebyshev polynomials and typically real functions**, Stanislawa Kanas

We discuss the generalized Chebyshev polynomials $U_n(p,q; e^{i\theta})$ of the second kind defined by the generating function

$$\Psi^{(p,q)}(e^{i\theta}; z) = \frac{1}{(1 - pze^{i\theta})(1 - qze^{-i\theta})} = \sum_{n=0}^{\infty} U_n(p,q; e^{i\theta})z^n \quad (z \in \mathbb{D}),$$

where $\theta \in [-\pi, \pi], \ (p,q) \in \Delta = \{(p,q) : -1 \leq q \leq p \leq 1\}.$

Based on the class of generalized typically-real functions was defined by S. Kanas, A. Tatarczak [Constrained coefficients problem for generalized typically real functions, Complex Var. Elliptic Equ. 61.8 (2016): 1052–1063, and Generalized typically real functions, Filomat 30:7 (2016), 1697–1710] that is the class of functions analytic in the unit disk, and given by

$$f(z) = \frac{1}{2\pi} \int_0^{2\pi} \frac{z d\mu(\theta)}{(1 - e^{i\theta}pz)(1 - e^{-i\theta}qz)} \quad (z \in \mathbb{D}),$$

(15)
where $\mu(\theta)$ is the probability measure on $[0, 2\pi)$, and $-1 \leq p, q \leq 1$.

[Based on a joint paper with A. Tatarczak]

Recurrences in the hyperbolic Pascal triangle and pyramid, László Németh

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There are several generalizations of Pascal’s arithmetical triangle, one among them is the family of hyperbolic Pascal triangles. This new type is based on the hyperbolic regular mosaics denoted by Schläfli’s symbol $\{p, q\}$, where $(p-2)(q-2) > 4$. Each regular mosaic induces a so-called hyperbolic Pascal triangle, it is detailed only for regular squared mosaics $\{4, q\}$ in six articles written by H. Belbachir, L. Szalay and the author. Obviously, the classical Pascal’s triangle is connected to the Euclidean square mosaic $\{4, 4\}$. This new mathematical construction has several interesting properties which are similar to the original triangle or fundamentally different.

The 3-dimensional analogue of Pascal’s original triangle is the well-known Pascal’s pyramid (or more precisely Pascal’s tetrahedron). Its levels are triangles and the numbers along the three edges of the $n$th level are the numbers of the $n$th lines of Pascal’s triangle. Each number inside in any levels is the sum of the three adjacent numbers on the level above. Three articles of the author deal with 3- and 4-dimensional variations, the hyperbolic Pascal (cube) pyramids and simplex, which are based on the higher dimensional hyperbolic regular (hypercube) mosaics.

In our presentation we introduce the hyperbolic Pascal triangle and pyramid, we show especially the recurrence relations of the growing from layer to layer, the (alternating) sum of layers, the special directions, the connection of layers’ pattern sequence and the Fibonacci word.

A model for an anisotropic flexoelectric material with cubic symmetry, Amr R. El Dhaba

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In this study, the responsive-behavior of an anisotropic, elastic, flexoelectric material with cubic internal structure to mechanical loads is formulated and ex-
examined. The elastic deformation and spontaneous polarization patterns of these materials, as well as the mechanical properties are described. The first strain gradient theory is employed to describe flexoelectric phenomenon in dielectric materials. The equations governing the elastostatic equilibrium of flexoelectric materials under infinitesimal strain gradient regimes are derived depending on the elegant variational formulation introduced by Topuin, and Mindlin. Explicit analytical expressions of the displacement and spontaneous polarization for the plane strain problem of the rectangular domain subjected to external forces is derived using the so-called semi inverse method. The magnesium oxide MgO (magnesia) is used as an example of dielectric materials with cubic internal structure since it is widely used in thin-film substrate and fabrication of microwave devices. The results revealed that flexoelectric materials exhibit spontaneous polarization due to the elastic extensibility and elastic compressibility.

**On the pointwise iteration-complexity of a dynamic regularized ADMM with over-relaxation stepsize**, Max L. Goncalves maxlmg@ufg.br

In this paper, we extend the improved pointwise iteration-complexity estimation of a dynamic regularized alternating direction method of multipliers (ADMM) for a new stepsize domain. In this complexity analysis, the stepsize parameter can be chosen in the interval $(0, 2)$ instead of interval $(0, (1+\sqrt{5})/2)$. We illustrate, by means of a numerical experiment, that the enlargement of this stepsize domain can lead to better performance of the method in some applications. Our complexity study is established by interpreting this ADMM variant as an instance of a hybrid proximal extragradient framework applied to a specific monotone inclusion problem.

**Counterexample to Gronwall’s Conjecture**, Sergey I. Agafonov sergey.agafonov@gmail.com

We present a projectively invariant description of planar linear 3webs and prove that a counterexample to Gronwall’s conjecture cannot have two pencils of lines as two web foliations. For a non-hexagonal 3-web, we also introduce a family
of projective torsion-free Cartan connections, the web leaves being geodesics for each member of the family, and give a web linearization criterion.

**Chaos in piecewise smooth dynamical systems on two dimensional torus, Ricardo M. Martins and Durval J. Tonon** rmiranda@unicamp.br

This paper studies the global dynamics of piecewise smooth differential equations defined in the two dimensional torus in the case when the switching manifold breaks the manifold into two connected components. Over the switching manifold we consider the Filippov’s convention for discontinuous differential equations. The study of piecewise smooth dynamical systems over torus is common for maps and up to where we know this is the first characterization for piecewise smooth flows arising from solutions of differential equations. We provide conditions under generic families of piecewise smooth equations to get periodic and dense trajectories. Considering these generic families of piecewise differential equations, we classify the minimal sets of the flow and prove that a chaotic behavior appears. Global bifurcations are also classified.

**Laplacian solitons on nilpotent Lie groups,** Marina Nicolini marinicolini9@gmail.com

We investigate the existence of closed $G_2$-structures which are solitons for the Laplacian flow on nilpotent Lie groups. We obtain that seven of the twelve Lie algebras admitting a closed $G_2$-structure do admit a Laplacian soliton. Moreover, one of them admits a continuous family of Laplacian solitons which are pairwise non-homothetic and the Laplacian flow evolution on four of the Lie groups is not diagonal.

**The index of symmetry of noncompact spaces,** Silvio Reggiani reggiani@fceia.unr.edu.ar

We approach the problem of determining the index of symmetry of noncompact homogeneous spaces. On the one hand, we want to understand up to what
extent the structure theory for compact spaces holds in the noncompact case. On the other hand, a classification (in low dimensions) of noncompact spaces according to its (co-)index of symmetry is always desirable. We manage to compute the index of symmetry for 3-dimensional unimodular Lie groups with left invariant metrics, and we also obtain some results on 2-step nilmanifolds. More precisely, we show that the distribution of symmetry of a naturally reductive nilpotent Lie group coincides with the invariant distribution induced by the set of fixed vectors of the isotropy. This extends a known result on compact naturally reductive spaces. In addition, we extend to the noncompact case the result which says that the distribution of symmetry of a homogeneous space cannot be of codimension 1.

Effect of Probability Sampling Techniques on Gaussian Naive Bayes Classification for Neonatal Survival, Ezekiel Gyimah and Atinuke O. Adebani

In data sparse countries, statistics are not sufficiently complete to allow reliable classification of neonatal survival, therefore appropriate sampling methods on existing data are required for proper prediction of neonatal mortality and survival. These methods should result in unbiased, accurate and precise estimates of population parameters. This study focused on the effect of probability sampling techniques – simple random sampling, stratified sampling and systematic sampling – on the performance of Gaussian Naive Bayes classification algorithm for predicting neonatal survival. Performance evaluation for the different sampling techniques applied was done using the 5-fold cross validation, leave-one-out cross validation and balanced error rate procedures. The study considered 520 neonatal data (260 mortality and 260 survival) sampled from 3869 discharges (survivals) and 858 deaths available from Maternity and Baby Unit (MBU) at a Teaching Hospital in Ghana, using the three sampling techniques. After pre-processing the data, three out of an initial seven predictors were used in the analysis – age of baby, maternal age and parity. Results obtained showed the systematic sampling technique as the best performer for predicting neonatal survival across all performance evaluation procedures.
On the Extension of Computable Real Functions, Walid Gomaa and Mathieu Hoyrup walid.gomaa@ejust.edu.eg

We investigate interrelationships among different notions from mathematical analysis, effective topology, and classical computability theory. Our main object of study is the class of computable functions defined over an interval with the boundary being a left-c.e. real number. We investigate necessary and sufficient conditions under which such functions can be computably extended. It turns out that this depends on the behavior of the function near the boundary as well as on the class of left-c.e. real numbers to which the boundary belongs, that is, how it can be constructed. Of particular interest a class of functions is investigated: sawtooth functions constructed from computable enumerations of c.e. sets.

Simultaneous extension of equivariant maps, Sergey Antonyan and Lili Zhang antonyan@unam.mx

Let $X$ be a metrizable space, $A$ a closed subset of $X$, and $L$ a locally convex topological vector space. Let $C(X, L)$ denote the vector space of continuous functions from $X$ into $L$, and similarly for $C(A, L)$. We equip these function spaces with the compact-open topology. The famous Dugundji extension theorem asserts that for every $f \in C(A, L)$ there exists $\Lambda(f) \in C(X, L)$ such that $\Lambda(f)|_A = f$ and $\text{Im} \Lambda(f) \subset \text{conv} (\text{Im} f)$. In 1953, E. Michael and R. Arens independently observed that the map $\Lambda : C(A, L) \to C(Z, L)$ constructed by Dugundji is, in fact, a linear homeomorphic embedding. In this talk we will discuss how to extend this result to the category of $G$-spaces. We will prove that an analogous equivariant result is true when the acting group $G$ is compact Lie. By an example, we will show that the result may fail to be true when $G$ is a compact non-Lie group.

The speaker thanks the Department of Mathematics of the Xi’an Technological University, where this joint research with Lili Zhang was completed.

Composition formulae for some secondary and tertiary homotopy operations, Howard J. Marcum marcum.1@osu.edu
The classical triple Toda bracket homotopy operation \( \{a, b, c\} \) can be defined in any 2-category \( C \) with zeros and takes its values in an obvious automorphism group. Hardie, Kamps, Marcum, Oda and others have developed additional operations in such an abstract setting, included among which are the box brackets, matrix Toda brackets and 2-sided matrix Toda brackets. The above mentioned automorphism group can be realized by postulating the existence of a suspension 2-functor \( \Sigma \) on \( C \). If furthermore the 2-category \( C \) admits a double mapping cylinder 2-functor \( M \) related to \( \Sigma \) then it admits a theory of extensions and coextensions, and accordingly tertiary operations can be defined. In this setting we here give definitions of several new tertiary operations by using a double box construction based on homotopy coherent cubes (rather than squares). We focus particular attention on a tertiary operation called the box quaternary operation.

Specific computations of secondary homotopy operations often depend upon inclusion formulae, and especially equalities, that occur after composition with a suitable element. Among the most useful of the latter are Toda’s formula
\[
\alpha_0 \circ \{ \alpha_1, \alpha_2, \alpha_3 \} = \{-\{\alpha_0, \alpha_1, \alpha_2\} \circ \Sigma \alpha_3 \text{ for Toda brackets and its generalization for matrix Toda brackets (due to Hardie, Marcum and Oda).}
\]
Our main results provide corresponding, and previously unknown, equalities for certain tertiary operations.

Residually (post) liminal \( C^* \)-algebras, Massoud Amini mamini@modares.ac.ir

The notion of residually CCR (liminal) and residually GCR (postliminal) \( C^* \)-algebras is defined and discussed. We prove perseverance of these properties under Morita equivalence (\( \sigma \)-unital case), minimal tensor products, and taking sub-algebras or crossed products by actions of compact groups. We also show that every residually CCR \( C^* \)-algebra is quasi diagonal (QD).

We define analogous notions for topological groups and show that the full group \( C^* \)-algebra \( C^*(G) \) is residually CCR (GCR), iff \( G \) is so. Also \( G \) is residually CCR iff the reduced group \( C^* \)-algebra \( C_r^*(G) \) is the quotient of a residually CCR \( C^* \)-algebra, which is in turn a quotient of \( C^*(G) \). Finally, we defined similar notions for representations and prove that all (cyclic) representations are residually CCR (GCR) if and only if the algebra itself is so.
This is a joint work with my PhD student Saied Jamali.

Hamilton Decomposition of Knodel Graphs and Fibonacci Graphs, Rejikumar Karunakaran and Jasmine Mathew rkkresearch@yahoo.in

Graph decomposition is the partitioning of the edge set. If each partite edge set induces a Hamilton cycle, the decomposition is Hamilton decomposition. The degree regularity of the given graph is necessary for the existence of Hamilton decomposition. When the degree of each vertex $d$ is even, the graph can be decomposed into disjoint Hamilton cycles, whose union is the given graph. If the degree of each vertex is odd, then each partite edge set induces a Hamilton cycle and one perfect matching. The history of Hamilton decomposition had begun in 1890’s, by Walecki’s construction for decomposing complete graphs. We study the Hamilton decomposability of Knodel graphs and Fibonacci Graphs, which are good topologies in the construction of optimal gossiping and broadcasting networks. Authors decompose knodel graph $W_{d,2n}$ whenever $n$ is odd and Fibonacci graph $F_{d,2n}$ whenever $(n, F(r)) = 1$, for $1 \leq r \leq F^{-1}(n)$ and $r$ is even into Hamilton cycles.

Spectrum of Gallai Graph of Some Graphs, Jeepamol J. Palathingal, Gopalapillai Indulal and Aparna L. S jeepamoljp@gmail.com

The Gallai graph $\Gamma(G)$ of a graph $G$, has the edges of $G$ as its vertices and two distinct vertices are adjacent in $\Gamma(G)$ if they are adjacent edges in $G$, but do not lie on a triangle. In this paper we find the adjacency spectrum of Gallai graph of some graphs derived from simple graphs by certain operations, the characteristic polynomial for some graphs and exhibit the adjacency matrix of some other graphs.

Fixed points of generalized weakly contractive mappings, Maria Samreen maria.samreen@hotmail.com

In this paper we establish fixed point theorems for generalized weakly contrac-
tive mappings using graph theoretic approach. Consequently, we also obtain fixed points results for cyclic contractions and \( \alpha \) type contraction mappings. Our theorems generalize, extend and unify some recent results by Harjani and Sadarangani and those contained therein.

**Local distality and distal expansions of stable theories**, Artem Chernikov
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I will give an overview of some recent interactions between Shelah’s classification in model theory and extremal combinatorics for restricted families of graphs. The class of distal theories captures those NIP theories which have no non-trivial stable parts (examples are given by the o-minimal theories, p-adic fields and the field of transseries). While distality of a theory is not preserved under reducts, my recent work with Starchenko and others demonstrates that graphs definable in a reduct of a distal theory satisfy many strong combinatorial properties previously known in the special case of semialgebraic graphs (e.g. the strong Erdos-Hajnal property and strong Szemerédi-Trotter type bounds). Motivated by this phenomena, we develop a notion of local distality and discuss the question of which stable theories admit distal expansions.

**Secure Domination of Some Networks**, Chithra M. R. and Manju K. Menon
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A physical connection between the different components of a parallel system is provided by an interconnection network. The topological structure of a network can be described by a connected graph \( G = (V, E) \) where \( V(G) \) is a set of nodes to be connected and \( E(G) \) is a set of direct communication links between the nodes. Many graph theoretic parameters are used to study the efficiency and reliability of an interconnection network. A set \( S \subseteq V(G) \) is said to be secure if the security condition, for every \( X \subseteq S, |N[X] \cap S| \geq |N[X] - S| \) holds. The cardinality of a minimum secure set in \( G \) is called the security number of \( G \). A set \( S \subseteq V(G) \) of vertices in a graph \( G \) is called a dominating set if every \( v \in V(G) \) is either an element of \( S \) or is adjacent to an element of \( S \). Now, a set \( S \subseteq V(G) \) is secure
dominating, if it is both secure and dominating. The secure domination number of $G$, is the minimum cardinality of a secure dominating set in $G$. In the current era, security is definitely a desirable property for the interconnection networks and hence these type of study has wide applications. In this paper, we have studied the security number and secure domination number of some networks.

From geometry and architecture to building information modelling BIM, Hassan Ait Haddou hassan.aithaddou@montpellier.archi.fr

Let $M$ be a $n$-dimensional closed manifold. The aim of this work is to introduce and give some properties around architectural geometry which is an important generalization of the basic geometry.

The purpose of this work is to give an overview of recent research in geometry applied to architecture by using computer tools. We have developed in our recent research new models to analyze and assess environmental impacts over the entire life cycle of product in the $n$-dimensional space $M$.

Mathematical models and the results obtained in the context of research projects in the field of architectural geometry are not always ”interoperable” because simulators used are often closed and not open source. The question that then arises is how to produce a tool that allows the flow of information between the various simulators based on complex geometry that are complementary but not integrated. This kind of problem is illustrated by the introduction of what is called the digital model or Building Information Modeling BIM.

The multi-criteria decision-making process has become a real challenge in the field of architecture and urban planning in general, and sustainable energy management in particular. These techniques provide solutions to problems related to the optimization of the architectural project, from design to manufacturing. Thus, several mathematical models and methods are used in the process of architectural design.

The boundary representation ($B-rep$) and Constructive Solid Geometry ($CSG$) approaches are used to compute a new complex architecture forms. In this presentation, we first introduce a new BIM design application that supports parametric relations to complex curves and surfaces (example of Nonuniform rational basis spline ($NURBS$)). We then show some examples of the application.
of geometry shapes in the domain of architectural design and urban planning.

**Generators and closed classes of groups**, Ramon Flores and Jose L. Rodriguez

We show that in the category of groups, every singly-generated class which is closed under isomorphisms, direct limits and extensions is also singly-generated under isomorphisms and direct limits, and in particular is co-reflective. We also establish several new relations between singly-generated closed classes. With 2-density exactly 2, which is determined by triangle subgraphs only, we show that the threshold probability for Maker’s win can be anywhere between 9/5 and 2.

**Equations with Order Degeneracy and Application to Cusped Piezo-Electric Prismatic Shells**, George V. Jaiani

Cusped prismatic shells considered as 3D bodies may have non-Lipschitz surfaces as the boundaries and their thicknesses may vanish at the edge. Using I. Vekua’s dimension reduction method, complexity of the 3D domain occupied by the body will be transformed in the degeneracy of the order of the 2D governing equations of the constructed hierarchy of 2D models on the boundary of the 2D projection of the 3D bodies under consideration.

Consideration of boundary value problems (BVP) and initial boundary value problems (IBVP) for elastic cusped prismatic shells leads to investigation of non-classical BVPs and IBVPs for the governing elliptic and hyperbolic systems of equations of the second order with order degeneracy on the boundary of the domain under consideration in the case of the two spatial variables. Initial conditions for the so called weighted mathematical moments of displacements remain classical, while the boundary conditions (BC) for them are nonclassical, in general. It means that in certain cases the Dirichlet BCs should be replaced by the Keldysh BCs (i.e. some parts of the boundary, where the order of the equations degenerate, should be freed from the BCs) and in certain cases weighted BCs should be set.

The present talk deals with hierarchical models of cusped piezo-electric pris-
matic shells. It is proved that like the weighted mathematical moments of the
displacements the BCs for the mathematical moments of the electric potential are
nonclassical. Modifications of methods developed for the degenerate equations
and systems of equations are used.

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tic piezoelectric structures].

A Vaccination Game, José Martins and Alberto A. Pinto jmmartins@ipleiria.pt

In this work, we consider a vaccination game where people have to choose
between to vaccinate or not to vaccinate, depending on the morbidity risks from
the vaccine and the morbidity risks from the infection. We introduce the evolu-
tionary vaccination dynamics for the reinfection SIRI model and we prove that it
is bistable. The bi stability of the evolutionary dynamics indicates that the damage
provoked by false vaccination scares can be much higher and much more persist-
tent than in the SIR model

The complete associated groups of quasigroups, Parascovia Syrbu syrbuviv@yahoo.com

The isotopy (a generalization of the isomorphism) and parastrophy (the in-
verses of a quasigroup operation) are natural transformations of quasigroups. The
product of an isotopy and a parastrophy of a quasigroup is called an isostrophy.
The right (left, middle) translation of a quasigroup \((Q, \cdot)\), by an element \(a \in Q\),
is the mapping: \(R_a(x) = x \cdot a\) (respectively, \(L_a(x) = a \cdot x\), \(J_a(x) = x \setminus a\)), where
\(\setminus\) is the left division in \((Q, \cdot)\). The group generated by all left and all right trans-
lations of a quasigroup \((Q, \cdot)\) is called the multiplication group (or the associated
group) of \((Q, \cdot)\) and represent an important tool in the theory of quasigroups. Rele-
vant results in this area are obtained by Bruck, Albert, Smith, Belousov, Kepka,
Ihringer, Niemanmaa, Drapal et al.) It is well known that the multiplication groups
of isotopic loops are isomorphic.
We consider the group generated by all left, right and middle translations of a quasigroup, which is called the complete associated group, its action in the isostrophic loops. In particular, we prove that the complete associated groups of isostrophic loops (principal isostrophic loops) are isomorphic (respectively, coincide).

The stabilizer of an element \( h \in Q \) in the (complete) associated group of a quasigroup \((Q, \cdot)\), is called the (complete) inner mapping group of \((Q, \cdot)\), with respect to \( h \). It is known the major role of the inner mapping group, for example, in the theory of normal subquasigroups (subloops). The inner mapping group of a quasigroup (loop) is generated by three families of mappings. We give a set of generators (consisting of five families of mappings) of the complete inner mapping group of a loop. If the loop is power—associative than its complete inner mapping group is generated by four families of mappings.

**The Haagerup–Pisier–Ringrose inequality**, Mohammad Sal Moslehian
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The Haagerup–Pisier–Ringrose inequality states that if \( \Phi \) is a bounded linear map from a \( C^* \)-algebra \( \mathcal{A} \) into a \( C^* \)-algebra \( \mathcal{B} \), then

\[
\left\| \sum_{j=1}^{n} \{\Phi(A_j)^*\Phi(A_j) + \Phi(A_j)\Phi(A_j)^*}\right\| \leq K \|\Phi\|^2 \left\| \sum_{j=1}^{n} \left( A_j^*A_j + A_jA_j^* \right) \right\|
\]

holds for some \( K \) and for any finite family \( \{A_1, \cdots, A_n\} \) of elements of \( \mathcal{A} \). In this talk, we present several versions of the Haagerup–Pisier–Ringrose inequality involving unitarily invariant norms and unital completely positive maps. Among other things, we show that if \( \mathcal{J} \) is the ideal of \( \mathbb{B}(\mathcal{H}) \) associated to a unitarily invariant norm \( ||| \cdot ||| \), \( \Phi : \mathcal{A} \to \mathcal{B} \) is a bounded linear map between \( C^* \)-algebras, \( A_1, \cdots, A_n \in \mathcal{A} \) are positive such that \( \Phi(A_j) \)'s commute for \( 1 \leq j \leq n \) and \( X_1, \cdots, X_n \in \mathcal{J} \), then

\[
\left\| \sum_{j=1}^{n} \{\Phi(A_j)X_j\Phi(A_j)^* + \Phi(A_j)^*X_j\Phi(A_j)\} \right\| \leq 4|||X_1 \oplus \cdots \oplus X_n||| \|\Phi\|^2 \left\| \sum_{j=1}^{n} A_j^2 \right\| .
\]
Analysis and numerical study of a mixed formulation of a tow membranes problem, El Bekkaye Mermri and Mohammed Bouchlaghem e.mermri@ump.ac.ma

Let $\Omega \subset \mathbb{R}^n$ be an open bounded domain with a smooth boundary $\partial \Omega$, $(f_1, f_2)$ be an element of $L^2(\Omega) \times L^2(\Omega)$ and $(g_1, g_2)$ be an element of $H^{1/2}(\partial \Omega) \times H^{1/2}(\partial \Omega)$. Set

$$K := \left\{(v_1, v_2) \in H^1_{g_1}(\Omega) \times H^1_{g_2}(\Omega) : v_1 \geq v_2 \text{ a.e. in } \Omega \right\},$$

where

$$H^1_{g_i}(\Omega) := \{v \in H^1(\Omega) : v = g_i \text{ on } \partial \Omega\}, \quad i = 1, 2.$$ 

We consider the following variational inequality problem:

$$\begin{cases}
\text{Find } u = (u_1, u_2) \in K \text{ such that } \\
\sum_{i=1}^{2} \int_{\Omega} \nabla u_i \cdot \nabla (v_i - u_i) \, dx + \sum_{i=1}^{2} \int_{\Omega} f_i (v_i - u_i) \, dx \geq 0 \quad \forall (v_1, v_2) \in K.
\end{cases}$$

This problem is called the two membranes problem, where the solutions $u_1$ and $u_2$ describes the displacement of two uniform elastic membranes, respectively, subjected to uniform extern forces. The problem consists of finding the equilibrium position of the two membranes.

In this paper, we introduce a new mixed formulation of the two membranes problem based on appropriate variational inequality of the second kind and the notion of the subdifferential of a convex continuous function of which the subdifferential $\mu$ leads to the characterization of the coincidence set, which is one of the unknowns of the problem. The reformulated problem is transformed into a saddle point problem:

$$\begin{cases}
\text{Find } (w, \mu) \in \left[H^1_0(\Omega)\right]^2 \times C \text{ such that } \\
\mathcal{L}(w, \lambda) \leq \mathcal{L}(w, \mu) \leq \mathcal{L}(v, \mu) \quad \forall (v, \lambda) \in \left[H^1_0(\Omega)\right]^2 \times C,
\end{cases}$$

where $\mathcal{L}$ is an appropriate Lagrange operator, $C$ a closed convex set and $(u_1, u_2) = (w_1, w_2) + (g_1, g_2)$. 

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Then we present an iterative method to solve the problem and we prove the convergence of the approximate solutions to the exact one. To validate the theoretical study we present some numerical examples.

**On generalized derivations in rings and Banach Algebras**, Nadeem ur Rehman
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Let \( R \) will be an associative ring, \( Z(R) \) the center of \( R \), \( Q \) its Martindale quotient ring and \( U \) its Utumi quotient ring. The center of \( U \), denoted by \( C \), is called the extended centroid of \( R \). For \( x, y \in R \) and we set \([x, y]_0 = x, [x, y]_1 = xy - yx\) and inductively \([x, y]_k = [[x, y]_{k-1}, y] \) for \( k > 1 \). The ring \( R \) is said to satisfy an Engel condition if there exists a positive integer \( k \) such that \([x, y]_k = 0 \) for all \( x, y \in R \). Notice that an Engel condition is a polynomial \([x, y]_k = \sum_{i=0}^{k} (-1)^i \binom{k}{i} y^i x y^{k-i} \) in non-commutative indeterminates \( x, y \). Recall that a ring \( R \) is prime if \( xRy = \{0\} \) implies either \( x = 0 \) or \( y = 0 \), and \( R \) is semiprime if \( xRx = \{0\} \) implies \( x = 0 \). An additive mapping \( d : R \rightarrow R \) is called a derivation if \( d(xy) = d(x)y + yd(x) \) holds for all \( x, y \in R \). In particular \( d \) is an inner derivation induced by an element \( q \in R \), if \( d(x) = [q, x] \) holds for all \( x \in R \). By a generalized inner derivation on \( R \), one usually means an additive mapping \( F : R \rightarrow R \) if \( F(x) = ax + xb \) for fixed \( a, b \in R \). For a such a mapping \( F \), it is easy to see that \( F(xy) = F(x)y + x[y, b] = F(x)y + xI_b(y) \). This observation leads to the following definition : an additive mapping \( F : R \rightarrow R \) is called generalized derivation associated with a derivation \( d \) if \( F(xy) = F(x)y + xd(y) \) for all \( x, y \in R \). In the present talk, we investigate the commutativity of \( R \) satisfying certain properties on some appropriate subset of \( R \). We also examine the case where \( R \) is a semiprime ring. Finally, as an application we obtain some range inclusion results of continuous or spectrally bounded generalized derivations on Banach algebras.

**Predictor-corrector p- and hp-versions of the finite element method for Poisson’s equation in polygonal domains**, Boniface Nkemzi and Stephanie Tanekou
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We consider boundary value problems for the Poisson equation on polygonal domains with general nonhomogeneous mixed boundary conditions and derive, on the one hand, explicit extraction formulas for the coefficients of the singularities. On the other hand, the formulas are used to construct efficient adaptations for the $h$-, $p$- and $hp$-versions of the finite element method for the numerical treatment. A priori error estimates show that the $h$-version of the finite element algorithm exhibits the same rate of convergence as it is known for problems with smooth solutions. However, the principal results of the present work are the robust exponential convergence results for the $p$- and $hp$-versions of the finite element method on quasiuniform meshes. In fact, it is shown that if the input data (source term and boundary data) are piecewise analytic, then with appropriate choices of conforming finite element subspaces $V_N$ of dimension $N \in \mathbb{N}$, the $p$- and $hp$-versions of the finite element algorithms on quasiuniform meshes yield approximate solutions $u_N$ to the exact solution $u$ that satisfy the estimates

$$
\|u - u_N\|_{H^1(\Omega)} \leq C_1 e^{-b_1 N^{\frac{3}{2}}} \quad \text{and} \quad \|u - u_N\|_{H^1(\Omega)} \leq C_2 e^{-b_2 N^{\frac{3}{2}}},
$$

respectively. Several numerical experiments are included to illustrate the practical effectiveness of the proposed algorithms. The results show that the theoretical error analyses are attained within the range of engineering accuracy.

Double Roman Domination in Product Graphs, Anu V and Aparna L. S
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Given a graph $G = (V, E)$, a function $f : V \rightarrow \{0, 1, 2, 3\}$ having the property that if $f(v) = 0$, then there exist $v_1, v_2 \in N(v)$ such that $f(v_1) = f(v_2) = 2$ or there exists $w \in N(v)$ such that $f(w) = 3$, and if $f(v) = 1$, then there exists $w \in N(v)$ such that $f(w) \geq 2$ is called a double Roman dominating function (DRDF). The weight of a DRDF $f$ is the sum $f(V) = \sum_{v \in V} f(v)$, and the minimum weight of a DRDF on $G$ is the double Roman domination number, $\gamma_{dR}(G)$ of $G$. In this paper, we study the double Roman domination number of Cartesian product graphs and strong product graphs. More precisely, we study the double Roman domination number of grid graphs.
Sensual Mathematics, Kirsi Peltonen Aalto University kirsi.peltonen@aalto.fi

We will describe some ideas and concrete openings at Aalto University to enhance interaction between science and arts. There is both a need and an opportunity for students to engage with modern mathematics not contained in the current curriculum. This includes students in not only traditional schools of science and engineering, but also programs in arts and economics. To respond to these needs, we are building a platform where not only students from diverse fields and different levels of education, but also teachers with different backgrounds, can share their ideas and views. The student exhibition Sensual Mathematics at Heureka Science Centre (May-Oct 2017) showed one way to share the beauty of mathematics for a broad audience. We provide a glimpse to the principles, content and outcomes of our learning environment. Current plans towards academic Aalto Math&Arts Minor consisting of transdisciplinary courses integrating mathematics with visual arts, design and architecture including collaboration with Emma Museum of Modern Art is discussed.

Geometrical and Analytical Properties of Chebyshev Sets in Riemannian Manifolds, Ronaldo F. de Lima ronaldo@ccet.ufrn.br

In this work, we study subsets of Riemannian manifolds known as Chebyshev sets, which are characterized by the existence of a well defined distance-realizing projection onto them. Important examples of Chebyshev sets are the souls of complete and noncompact manifolds with nonnegative sectional curvature, as introduced by J. Cheeger and D. Gromoll. Our approach relates analytical properties of the distance function to Chebyshev sets to their geometrical properties, giving particular attention to convexity. In this setting, we prove that a simple Chebyshev set $C$ of a complete connected Riemannian manifold $M$ of nonnegative curvature is totally convex. Moreover, if $C$ has empty boundary, then it is a submanifold of $M$ whose normal bundle is diffeomorphic to $M$. We also establish that in a complete connected Riemannian manifold $M$ of nonnegative Ricci curvature, the distance function to a simple Chebyshev set is subharmonic. Conversely, if the sectional curvature of $M$ is nonnegative and the distance function to a closed set
$C \subset M$ is subharmonic, then $C$ is a simple Chebyshev set.

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Lattices have been used in various sub-areas of information theory and codes. A full-rank lattice is a discrete additive subgroup of $R^n$, i.e., a set composed by all integer linear combination of $n$ linearly independent vectors in this space.

It has been shown that algebraic lattices, i.e., lattices constructed via the canonical embedding of an algebraic number field, provide an efficient tool for designing lattice codes and enable the computation of invariants such as density and minimum product distance, which are important in applications in error-correcting codes and in cryptographic schemes based on lattices. The construction using ideals in algebraic number fields generates the so-called ideal lattices or trace lattice, which are obtained by a scaled trace form.

We propose an algebraic construction of lattices of dimension $8n$ via orders in octonion algebras. We can define ideal lattices from octonion orders in the same way that we define ideal lattices from number fields.

By using our construction we obtain lattices of dimension $2^n$. An important infinite sequence of lattices in these dimensions is called Barnes-Wall lattices. This family include the densest packings known in dimensions 1, 2, 4, 8 and 16. In dimensions 32 and higher they are less dense than other known packings, but they are still interesting because they form one of the few infinite sequences of lattices where it is possible to do explicit calculations. Applying our construction we obtain rotated versions of Barnes-Wall lattices in dimensions 8, 16 and 32 via octonion algebras.

**A kind of hyperbolic dynamics created by pseudo-riemannian metrics**, Mohammadreza Molaei  
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In this talk we present the notion of hyperbolic fixed points for dynamical systems on pseudo-Riemannian manifolds via a distribution which has no any spacelike or timelike vector. Since the unit spheres in the pseudo-Riemannian
tangent spaces are not compact, then we have to use of a new norm on linear transformations of the tangent space at a point of the ambient manifold. We study the properties of this norm, and it’s effect to the notion of stability of dynamical systems.

Asymptotic behavior of the lower and higher order Boussinesq System of KdV-Type, Fernando Gallego, Roberto Capistrano and Ademir Pazoto

A family of Boussinesq systems has been proposed before, to describe the bi-directional propagation of small amplitude long waves on the surface of shallow water. In this talk, we present some recent results related to boundary stabilization of the generalized lower and higher order Boussinesq systems of KdV–type posed on a interval under the Dirichlet-Neumann boundary condition.

Firstly, we consider the fifth-order Boussinesq system, a Higher Order KdV-KdV Type:

\[
\begin{align*}
\eta_t + u_x - au_{xxx} + a_1(\eta u)_x + a_2(\eta u_{xx})_x + bu_{xxxxx} &= 0, \\
u_t + \eta_x - a\eta_{xxx} + a_1uu_x + a_3(\eta\eta_{xx})_x + a_4u_xu_{xx} + b\eta_{xxxxx} &= 0,
\end{align*}
\]

with \(a > 0, b > 0, a \neq b, a_1 > 0, a_2 < 0, a_3 > 0\) and \(a_4 > 0\), with the boundary conditions:

\[
\begin{align*}
\eta(0, t) = \eta(L, t) = \eta_x(0, t) = \eta_x(L, t) &= 0, \quad \text{in } (0, \infty), \\
u(0, t) = u(L, t) = u_x(0, t) = u_x(L, t) &= 0, \quad \text{in } (0, \infty), \\
u_{xx}(0, t) + \alpha_1\eta_{xx}(0, t) &= 0, \quad \u_{xx}(L, t) - \alpha_2\eta_{xx}(L, t) = 0, \quad \text{in } (0, \infty),
\end{align*}
\]

for \(\alpha_1, \alpha_2 \in \mathbb{R}^+_\). Our main result for the system (16) is to design a parameter family of feedback laws for which the solution of the associated linearized system are exponentially decreasing in the energy space.

Secondly, we deal with the local rapid exponential stabilization for a Boussinesq system of KdV-KdV type introduced as a model for the motion of small amplitude long waves on the surface of an ideal fluid, a Lower Order KdV-KdV
Type:

\[
\begin{cases}
\eta_t + w_x + w_{xxx} + (\eta w)_x = 0, & \text{in } (0, L) \times (0, +\infty), \\
w_t + \eta_x + \eta_{xxx} + w w_x = 0, & \text{in } (0, L) \times (0, +\infty), \\
\eta(x, 0) = \eta_0(x), \quad w(x, 0) = w_0(x), & \text{in } (0, L),
\end{cases}
\]  
(17)

with boundary condition

\[
\begin{cases}
\eta(0, t) = 0, \quad \eta(L, t) = 0, \quad \eta_x(0, t) = f(t), & t \in (0, \infty), \\
w(0, t) = 0, \quad w(L, t) = 0, \quad w_x(L, t) = g(t), & t \in (0, \infty)
\end{cases}
\]
(18)

where, the function \( f \) and \( g \) are given by a linear feedback control law

\[
(f(t), g(t)) = F[(\eta(\cdot, t), w(\cdot, t))], \quad t \in (0, \infty).
\]

Our goal is to build suitable integral transformations to get a feedback control law \( F \) that leads to the stabilization of the system. More precisely, we will prove that the solution of the closed-loop system decays exponentially to zero in the \( L^2(0, L) \)–norm and the decay rate can be tuned to be as large as desired if the initial data is small enough.

**A strong convergence test by microlocal defect distributions**, Jelena Aleksic 

[Email]

Microlocal defect distributions (also called H-distributions) are generalization of H-measures. An H-distribution is associated to a pair of weakly convergent sequences in dual spaces. Initially, H-measures are prescribed to a \( L^2 - L^2 \)-pair. Then, H-distributions are defined for \( L^p - L^q, \frac{1}{p} + \frac{1}{q} = 1 \), Sobolev, Bessel and Sobolev/Bessel-type spaces.

H-distribution acts on the product of two test functions, one dependent in space and another in frequency (Fourier) variable. Thus, sequences are viewed in the phase space perspective. Definition of this object involve Fourier multiplier operators and limit up to subsequences. Recent developments in hypoellipticity theory suggest the use of more general weight functions as multipliers.
Among many applications of these microlocal tools we would like to emphasize possibility of testing strong convergence of weakly convergent sequences in mentioned spaces. For example, a possible strong convergence of weakly convergent sequence in Sobolev space can be obtained if for all weakly convergent sequences in dual space the corresponding H-distribution is zero. Thus, possible strong convergence of weakly convergent sequence is tested on all weakly convergent sequences in dual space. Involving theory of pseudo-differential operators, the space for testing strong convergence became smaller.

Moreover, these results can be applied to sequence of approximated solutions to different types of differential equations.

On a lattice map of Anderson t-motives, Dmitry Logachev logachev94@gmail.com

Let $M$ be an uniformizable Anderson t-motive of dimension $n$ and rank $r$ having the nilpotent operator $N = 0$. We can associate it a lattice $L = \mathbb{F}_q[\theta]^r$ in $\mathbb{C}_\infty^n$. Hence, we have a lattice map from the set of uniformizable Anderson t-motives to the set of lattices (both up to an isomorphism).

The naive dimensions of the sets of pure uniformizable t-motives of dimension $n$ and rank $r$, as well as of the sets of the above lattices, are $n(r - n)$, hence we can conjecture that the lattice map for pure t-motives is a near-isomorphism. This is really so (the map is an isomorphism) for $n = 1$ (Drinfeld). The duality theory shows that pure t-motives of dimension $n = r - 1$ are all uniformizable, and the lattice map for them is an inclusion whose image is exactly the set of lattices having dual (almost all lattices of dimension $r - 1$ and rank $r$ satisfy this condition).

I shall give an infinitesimal result supporting the isomorphism conjecture. Any lattice of dimension $n$ and rank $r$ is characterized by its Siegel matrix $S$ of size $(n, r - n)$ with entries in $\mathbb{C}_\infty$. We consider the case $r = 2n$ and $S = \omega I_n$ for a fixed $\omega \in \mathbb{F}_{q^2} - \mathbb{F}_q$. The corresponding Anderson t-motive is the $n$-th direct power of the Carlitz module over $\mathbb{F}_{q^2}$. We shall show that infinitesimally, we have an isomorphism of the lattice map. Method of the proof: we compare the actions of monodromy groups for two cases: (a) from the set of equations defining t-motives to the set of t-motives themselves, and (b) from the set of Siegel matrices to the
The result of the present paper gives that the size of a neighborhood, where we have an isomorphism, depends on an element of the monodromy group. We do not know whether there exists a universal neighborhood. Method of the proof: explicit solution of an equation describing an isomorphism between two $t$-motives by a method of successive approximations using a version of the Hensel lemma.

We shall also discuss possibilities of further research. For example, we shall show that conjecturally the purity condition is essential: for non-pure $t$-motives the lattice map has (conjecturally) a fiber of dimension $> 0$.

Large subgroups of simple groups with applications in group theory, combinatorics and geometry, Seyed Hassan Alavi S.H. Alavi, Department of Mathematics, Faculty of Science, Bu-Ali Sina University, Hamedan, Iran. alavi.s.hassan@gmail.com

Let $G$ be a finite group. A proper subgroup $H$ of $G$ is said to be large if the order of $H$ satisfies the bound $|H|^3 \geq |G|$. The problem of determining the large maximal subgroups of finite simple groups has a long history, with many applications. In this talk, we give a survey on the study of large maximal subgroups of (almost) simple groups and its applications to group factorisations, combinatorics and geometry. These subgroups arose from the study of triple factorisations $G = ABA$ with $A$ and $B$ non-trivial subgroups of $G$. Large maximal subgroups also appear in a classification of finite antiflag-transitive generalized quadrangles as well as in the study of flag-transitive and point-primitive designs.

Huppert’s conjecture and almost simple groups, Ashraf Daneshkhah adanesh@basu.ac.ir

Let $G$ be a finite group and $\text{cd}(G)$ denote the set of complex irreducible character degrees of $G$. Huppert conjectured that if $H$ is a finite nonabelian simple group such that $\text{cd}(G) = \text{cd}(H)$, then $G \cong H \times A$, where $A$ is an abelian group. In this talk, provide some examples that we cannot extend this conjecture to almost simple groups. Moreover, we show that if $G$ is a finite group and $H$ is an
almost simple group whose socle is $H_0$ is a sporadic simple group or $PSL(2, 2^f)$ with $f$ prime such that $\text{cd}(G) = \text{cd}(H)$, then there exists an abelian subgroup $A$ of $G$ such that $G/A$ is isomorphic to $H$. Based on our further investigation, we propose a conjecture that if $G$ is a finite group and $H$ is an almost simple group such that $\text{cd}(G) = \text{cd}(H)$, then there exists an abelian subgroup $A$ of $G$ such that $G/A$ is isomorphic to $H$.

Mathematical modelling of risk preferences with application to portfolio optimization, Cristinca Fulga The Bucharest University of Economic Studies fulga@csie.ase.ro

In this talk, we propose an alternative methodology for defining, measuring and optimizing risk that addresses some of the conceptual shortcomings of the Mean-Risk framework such as the disregard of investor attitude towards risk and implicit assumption of neutrality to loss aversion. The key in our proposed methodology is a risk measure called Expected Shortfall with Loss Aversion (ESLA) that, for continuous return distribution functions, can be represented in terms of the conditional expectation of the distribution tail, where the tail is determined by the critical return level characterizing the loss-averse investor. Our contributions can be summarized as follows. We begin by defining ESLA, then we show the relations of the proposed risk measure with Conditional Value at Risk (CVaR), and Lower Partial Moment of first order (LPM1), establish its properties, study the link with stochastic dominance of first and second order criteria, and discuss practical aspects regarding the calculation in the case of scenario-based portfolio optimization. Next, we describe the portfolio selection methodology in two stages in which investor’s loss aversion is fully taken into consideration: firstly, the investment opportunity set is determined (in our case, the Mean-ESLA efficient frontier), and secondly, one single preferred portfolio is selected from the efficient frontier. We consider three types of investors characterized by three different classes of utility functions with loss aversion. The empirical study is targeted on assessing the differences between the Mean-ESLA efficient frontier and the classical Mean-Variance, Mean-CVaR and Mean-LPM1 frontiers. We measure the loss of welfare incurred by using another model instead of the proposed one using a proximity index in the Expected Utility framework. Then, in order
to assess how much the portfolios differ in terms of their compositions, we use a
dissimilarity index based on the $l_1$ norm. We also were concerned by the role and
influence of loss aversion parameters values and constraints. A test was performed
to study how the models behave when using various constraints. Firstly, the usual
non-negativity constraint was replaced by a set of box constraints. As expected,
as a result of shrinking the interval of variation of each security and consequently
reducing the feasible region, the similarity increases. For the final test, it is as-
sumed that short selling is allowed. As we expect, in this case the values of the
dissimilarity indices increase compared with the other types of constraints. The
analysis shows that the proposed model is not dominated by any of the classical
models. Moreover, the optimal/efficient solutions of the Mean-ESLA model dif-
fer markedly from the classical models Mean-Variance, Mean-CVaR and Mean-
LPM1 as the dissimilarity indices show and, in terms of utility, Mean-ESLA is
frequently similar, but also sometimes preferred to classical models.

Two stage hyper-chaotic system based image encryption in wavelet packet
domain for wireless communication systems, Manish Kumar manish.math.
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In this work, we propose a new algorithm to encrypt color-image data using
a hyper-chaotic system in the wavelet packet domain. The proposed algorithm is
based on random sequence generated through Lorenz hyper-chaotic system using
three different sets of initial conditions. The whole algorithm is divided into three
stages —the Lorenz hyper-chaotic system is used in two different stages as before
and after the wavelet packet domain. The advantage of the intermediate stage
(i.e., wavelet packet domain) will provides high security to the algorithm since
all pixels are almost close to zero. The proposed encryption algorithm offers high
security and larger keyspace and occupies smaller key data volume. The proposed
scheme renders diffusion and confusion in 3D and 2D forms followed by Bit-XOR
operation in the wavelet packet domain. Performance and security analysis shows
the robustness and efficiency of the proposed algorithm.

N-expansive homeomorphisms with the shadowing property, Bernardo M.
We discuss the dynamics of $n$-expansive homeomorphisms with the shadowing property defined on compact metric spaces. For every $n \in \mathbb{N}$, we exhibit an $n$-expansive homeomorphism, which is not $(n-1)$-expansive, has the shadowing property and admits an infinite number of chain-recurrent classes. We discuss some properties of the local stable (unstable) sets of $n$-expansive homeomorphisms with the shadowing property and use them to prove that some types of the limit shadowing property are present. This deals some direction to the problem of non-existence of topologically mixing $n$-expansive homeomorphisms that are not expansive.

**Contact CR-submanifolds in $\mathbb{S}^7(1)$**, Mirjana Djoric, Marian I. Munteanu and Luc Vrancken mdjoric@matf.bg.ac.rs

A well-known notion for submanifolds $M$ of Sasakian manifolds $\tilde{M}(\phi, \xi, \eta, \tilde{g})$ is the notion of a contact CR-submanifold, the analogue of CR-submanifolds in Kählerian manifolds. These are submanifolds for which the structure vector field $\xi$ is tangent to the submanifold $M$ and for which the tangent bundle can be decomposed as $T(M) = H(M) \oplus E(M) \oplus \mathbb{R}\xi$, where $H(M)$ is invariant with respect to $\phi$ and $E(M)$ is anti-invariant with respect to $\phi$. Since such submanifolds can never be totally geodesic, we study those submanifolds which are as close as possible to totally geodesic. We present a complete classification of four-dimensional contact CR-submanifolds in $\mathbb{S}^5(1)$ and $\mathbb{S}^7(1)$ whose second fundamental form restricted to $H(M)$ and $E(M)$ vanishes identically.

This is a joint work with M.I. Munteanu and L. Vrancken.

**String Topological Robotics**, My Ismail Mamouni CRMEF Rabat, Rabat, Morocco mamouni.myismail@gmail.com

We aim to link two well known theories; namely the string topology (founded by M. Chas and D. Sullivan in 1999) and the topological robotics (founded by M.
Farber some few years later, in 2003). For our purpose, we consider $G$ a compact Lie group acting on a path connected $n$-manifold $X$. The relevance of the Lie group $G$ will be discussed through the paper. On the set $\mathcal{MLP}(X)$ of the \textit{loop motion planning algorithms}, we define a kind of a string loop motion planning product, which endows the shifted homology $H_*(\mathcal{MLP}(X)) := H_{*+2n}(\mathcal{MLP}(X))$, with a structure of a graded commutative and associative algebra structure. We show after that it yields a structures of Gerstenhaber and Batalin-Vilkovisky algebras.

\textbf{MHD Ethanol Boundary Layers Due to a Point Sink with Variable Viscosity and Prandtl Number,} A T Eswara, Venkataseshaiah B, Roopadevi K N and Sarada Pakala \texttt{ateswara@gmail.com}

Viscosity is a physical property of fluids and it describes fluid’s internal resistance to the flow. Majority of the boundary layer research considers the flow of fluids with constant viscosity. However, in certain situations it is incorrect to treat viscosity as constant as it varies with distance, pressure and temperature. Therefore, it is highly desirable to take variable viscosity in momentum as well as energy equations. Further, Prandtl number is a function of viscosity, and as viscosity varies with temperature across the laminar boundary layer, the Prandtl number varies, too. The present work considers the influence of temperature-dependent viscosity and Prandtl number on the steady, laminar MHD flow (of ethanol) due to a point sink. The parabolic partial differential equations governing the axisymmetric boundary layer flow are transformed into non-dimensional form by a group of similarity transformations. The resulting system of coupled non-linear ordinary differential equations is solved using an implicit finite difference scheme in combination with the quasilinearization technique. Computations are performed to obtain the results for the dimensionless velocity, temperature, skin friction and heat transfer for various parametric conditions and, numerical results are discussed using graphical approach. It is revealed that skin friction factor and the heat transfer rate significantly influenced by the temperature-dependent viscosity as well as Prandtl number. This study exposes the fact that when the working fluid is sensitive to the temperature, the effect of variable viscosity and Prandtl number have to be taken into the consideration in order to predict the skin friction and heat transfer rate, accurately.
Behind the Scenes of the 1950 and 1958 Fields Medals., Michael J. Barany michael@mbarany.com

First presented in 1936, the Fields Medal quickly became one of mathematicians’ most prestigious, famous, and in some cases notorious prizes. Because its deliberations are confidential, we know very little about the early Fields Medals: how winners were selected, who else was considered, what values and priorities were debated. Two newly identified collections of correspondence, from the 1950 and 1958 deliberations respectively, allow the first detailed historical examination of how early Fields Medal committees interpreted their mandate, assembled and narrowed pools of candidates, and reached consensus over whom to recognize. Together with a wide selection of isolated letters from committee members, these materials show how age limits and related considerations emerged as ways to narrow the field of nominees and to justify omissions retrospectively, and that these were subject to considerable informal judgment about factors including career stage and prior recognition. Committees deliberately avoided using the Fields Medal to honor those they believed were already especially intellectually and professionally prominent. They sometimes discussed candidates’ mathematical work in general terms, but did not debate specific intellectual criteria in detail. While the early Fields Medals are often seen retrospectively as evidence of young mathematicians’ leading place in the discipline, these findings justify a more measured view of the prize’s place in mid-century mathematics.

Different Types of Information about Function in Approximate Calculation of Triple Integrals of Rapidly Oscillating Functions, Olesia P. Nechuiviter olesya@email.com

Report is dedicated to the improvement of mathematical models of digital signal processing and imaging by the example of constructing cubature formulas of approximate calculation of integrals of highly oscillatory functions of three variables. The feature of the proposed cubature formulas is using the input information about function or as a set of traces of function on planes or a set of traces of function on lines and of course as a set of values of the function in the points. The main advantages of methods are high exactness of approximation, the opportunity
to decrease the amount of information about function during the calculation.